MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXIII. APRIL 11, 1913. No. 6

Major E. H. HILLS, C.M.G., F.R.S., PRESIDENT, in the Chair.

E. A. Fath, Director of the Smith Observatory, Beloit College, Beloit, Wisconsin, U.S.A.;
Rev. George Harold Lancaster, M.A., 6 Dartmouth Road, Brondesbury, N.W.;
Maurice Egerton Leigh-Hunt, Queen's Hill, Hertford, Herts;
Rev. James Marchant, Lochnagar, Eden Bridge, Kent; and
William Alfred Parr, 19 Rosslyn Hill, Hampstead, N.W.,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

William Herbert Henry Brookes, Principal Observer at the National Physical Laboratory, Ewelme, Claremont Road, Teddington, S.W. (proposed by F. W. Dyson).

Dr. Robert G. Aitken, Lick Observatory, Mount Hamilton, California, U.S.A.;
Professor Fritz Cohn, K. Astronomisches Rechen-Institut, Berlin; and
Professor Alfred Wolfer, Ph.D., Sternwarte, Zurich, Switzerland,

were proposed by the Council as Associates of the Society.

Sixty presents were announced as having been received since the last Ordinary meeting.

The Distribution in Space of the Stars of Carrington's Circumpolar Catalogue (Second Paper). By F. W. Dyson, LL.D., F.R.S., Astronomer Royal.

1. The distribution of the components of the proper motions of the stars of Carrington's Circumpolar Catalogue in a direction perpendicular to the solar motion was discussed in the last number of the *Monthly Notices*. In this paper the proper motions in the direction of the solar motion will be considered.

2. The linear velocities of the stars in a component in which there is no systematic streaming were taken as distributed according to the law $\frac{h}{\sqrt{\pi}}e^{-h^2v^2}$. When there is a stream velocity V, whether arising from the solar motion or a general drift of the stars themselves, the frequency law becomes $\frac{h}{\sqrt{\pi}}e^{-h^2(v-V)^2}$. Out of N stars the number $N\frac{h}{\sqrt{\pi}}e^{-h^2(v-V)^2}dv$ have velocities between v and $v+dv$. Assuming the law of distance distribution of the previous paper, the number of them between the distances r and $r+dr$ is

$$N2k^2r\,e^{-k^2r^2}dr\,.\,\frac{h}{\sqrt{\pi}}e^{-h^2(v-V)^2}dv \quad . \quad . \quad . \quad (1)$$

Let v be the component of proper motion corresponding to a component of velocity v, so that $rv=v$; then the number of stars with proper motions v to $v+dv$ is

$$Nh\frac{dv}{\sqrt{\pi}}\int_0^\infty 2k^2r^2\,\mathrm{Exp.}\{-(k^2+h^2v^2)r^2+2hvhVr-h^2V^2\}dr\,.$$

Let

$$\left.\begin{aligned}(k^2+h^2v^2)^{\frac{1}{2}}r-h^2vV(k^2+h^2v^2)^{-\frac{1}{2}}&=x\\ hv&=k\tan\theta\end{aligned}\right\} \quad . \quad . \quad (2)$$

And

The number of proper motions between v and $v+dv$

$$=N\frac{2}{\sqrt{\pi}}\cos\theta\,d\theta\,\mathrm{Exp.}(-h^2V^2\cos^2\theta)\int_{-hV\sin\theta}^{\infty}(x+hV\sin\theta)^2\,.\,e^{-x^2}dx$$

$$=N\frac{2}{\sqrt{\pi}}\cos\theta\,d\theta(\tfrac{1}{2}+h^2V^2\sin^2\theta)\,.\,\mathrm{Exp.}(-h^2V^2\cos^2\theta)\int_{-hV\sin\theta}^{\infty}e^{-x^2}dx$$

$$+N\frac{1}{\sqrt{\pi}}\sin\theta\cos\theta\,d\theta\,.\,e^{-h^2V^2}\,.\,hV \quad . \quad . \quad (3)$$

$$=d\theta\,.\,N\frac{1}{\sqrt{\pi}}\,.\,\frac{d}{d\theta}\left\{\sin\theta\,.\,\mathrm{Exp.}(-h^2V^2\cos^2\theta)\int_{-hV\sin\theta}^{\infty}e^{-x^2}dx\right\}$$

3. Thus the number of proper motions from o to $+v$

$$= N\frac{1}{\sqrt{\pi}} . \sin\theta \, \text{Exp.}(-h^2V^2\cos^2\theta) \int_{-hV\sin\theta}^{\infty} e^{-x^2}dx \quad . \quad (4)$$

The number between $-v$ and $+v$

$$= N \sin\theta \, \text{Exp.}(-h^2V^2\cos^2\theta) \quad . \quad . \quad (5)$$

and the excess of the number between o and $+v$, over that between o and $-v$

$$= N \sin\theta \, . \, \text{Exp.}(-h^2V^2\cos^2\theta) \, . \, \frac{2}{\sqrt{\pi}} \int_{o}^{hV\sin\theta} e^{-x^2} . dx \quad . \quad (6)$$

When v is large θ approaches $\frac{\pi}{2}$, and it readily follows from (5) and (6) that the sum of the number of proper motions $> +v$ and $< -v$

$$\text{is } N\frac{k^2}{h^2v^2}(\tfrac{1}{2} + h^2V^2) \quad . \quad . \quad . \quad (7)$$

and the difference is

$$N\frac{k^2}{h^2v^2}\left\{ (\tfrac{1}{2} + h^2V^2)\frac{2}{\sqrt{\pi}} \int_{o}^{hV} e^{-x^2}dx + \frac{hV}{\sqrt{\pi}} e^{-h^2V^2} \right\} \quad . \quad (8)$$

4. The formulæ of the last paragraph may be compared with the observations, both on Mr. Eddington's and Prof. Schwarzschild's laws of star streaming. In Mr. Eddington's theory h is taken as known from the cross motions, and two streams with different values of V are assumed; while on Prof. Schwarzschild's theory V is the component of the solar motion and h is to be found from the observations. In both cases it is convenient, on account of the accidental error to which the numbers are liable, to begin with the large proper motions where the error is insensible.

5. The following table summarises the numbers of large centennial proper motions out of 3735 given by the observations—

Limits of p.m.	No. of Stars.	Limits of p.m.	No. of Stars.
$> 20''$	30	$< -12''$	18
$> 15''$	60	$< -8''$	31
$> 12''$	87	$< -6''$	54
$> 10''$	126		
$> 8''$	196		

Taking the number $> +12''$ or $< -12''$, we have on Schwarzschild's hypothesis

$$\frac{1}{2}\frac{k^2}{h^2} + k^2V^2 = \frac{105}{3735} . 12^2 \quad . \quad . \quad . \quad (9)$$

and $Vk = \frac{V}{\sqrt{\pi}}\overline{\pi} = \frac{1''\cdot44}{\sqrt{\pi}}$, taking the parallactic motion of these stars as $1''\cdot44$ a century.

This gives $\frac{k}{h} = 2''{\cdot}61$; the cross proper motions gave $\frac{k}{h} = 1''{\cdot}44$; so the ratio of the diameters of Schwarzschild's ellipse in these two directions is 1 : 1·81.

The value of $h\mathrm{V}$ is $\frac{1{\cdot}44}{\sqrt{\pi}} \cdot \frac{1}{2{\cdot}61} = {\cdot}311$.

With these values of $\frac{k}{h}$ and $h\mathrm{V}$, the number of proper motions $> v = \frac{\mathrm{N}}{v^2} 3{\cdot}40 \{{\cdot}60 + {\cdot}37\}$, and the number $< -v = \frac{\mathrm{N}}{v^2}\{{\cdot}60 - {\cdot}37\}$.

For $v = 12''$, these numbers are 85 and 20, as compared with the observed numbers 87 and 18.

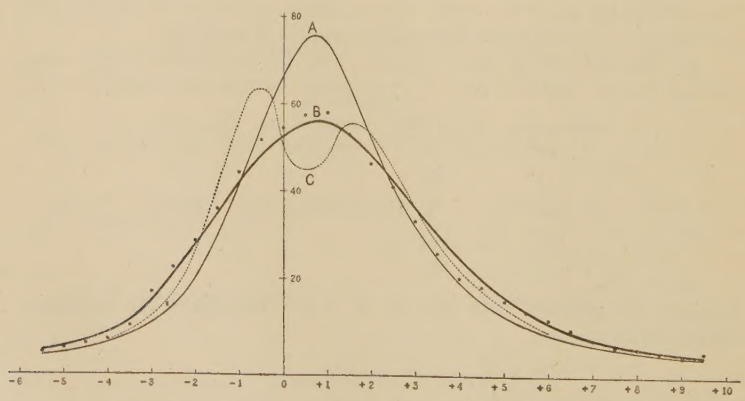
6. Starting from this first approximation, it was seen that too small a number of stars of small proper motion was obtained, and the value $\frac{k}{h} = 2''{\cdot}40$, which gives $h\mathrm{V} = {\cdot}339$, has been adopted.

The values of $\mathrm{N} \frac{1}{\sqrt{\pi}} \sin\theta \operatorname{Exp.}(-h^2\mathrm{V}^2 \cos^2\theta) \cdot \int_{-h\mathrm{V}\sin\theta}^{\infty} e^{-x^2} dx$, were calculated for values of θ, corresponding to intervals of $0''{\cdot}4$ in v, from $v = 0$ to $v = \pm 12''$.

Allowance was then made for a probable error of $1''{\cdot}0$ a century, numerically from the values of $-3''{\cdot}4$ to $+6''{\cdot}0$ of v, and outside these limits by Mr. Eddington's formula

$$v(x) = u(x) + \frac{\mathrm{D}^2}{4h^2} u(x) + \ldots$$

The results compared with the observations are shown in the accompanying diagram and table.



Distribution of Proper Motions resolved in direction of Solar Antapex.

A, single stream, with accidental velocities according to Schwarzschild's hypothesis; B, modified to allow for accidental error of $1''{\cdot}0$ per century in P.Ms.; C, two equal streams with accidental velocities the same in all directions. Observations denoted by dots.

Number of Stars with Centennial Proper Motion resolved in the direction of the Antapex between certain limits.

Centennial proper motion.	Formula not corrected.	Formula corrected for p.e.	Observation.	Formula – Observation.
$< -12''$	16	16	18	− 2
−12 to − 8	19	19	13	+ 6
− 8 to − 6	25	28	23	+ 5
− 6 to − 5	23	33	30	+ 3
− 5 to − 4	39	57	50	+ 7
− 4 to − 3	71	104	91	+13
− 3 to − 2	138	211	225	−14
− 2 to − 1	297	347	359	−12
− 1 to − 0	550	480	517	−37
0 to + 1	746	553	574	−21
+ 1 to + 2	643	528	530	− 2
+ 2 to + 3	411	425	410	+15
+ 3 to + 4	251	304	257	+47
+ 4 to + 5	149	201	182	+19
+ 5 to + 6	95	125	126	− 1
+ 6 to + 7	62	81	86	− 5
+ 7 to + 8	43	53	48	+ 5
+ 8 to + 9	31	36	34	+ 2
+ 9 to +10	23	27	36	− 9
−10 to +12	30	34	39	− 5
+12 to +15	26	27	27	0
+15 to +20	20	20	30	−10
$> +20$	27	27	30	− 3

Although the agreement is not perfect, it seems to be very satisfactory, as with only one constant h, the number of large positive, of large negative, and of small proper motions are fairly well satisfied, as well as the position of the maximum of the curve. Further, the value of h agrees with that found by Mr. Eddington (*B.A. Report*, 1909, p. 402) from the Groombridge stars for the ratio of the axes of the velocity-ellipse.

8. In Mr. Eddington's hypothesis of *two* star streams, the constant h which gives the spread of the stars is the same for both streams. With an equal number of stars in each of the two streams, the stars of large proper motion lead to the equation

$$N \frac{k^2}{h^2} \cdot \frac{1}{v^2} \left(\frac{1}{2} + h^2 \frac{{v_1}^2 + {v_2}^2}{2} \right) =$$

number of stars with proper motion $> +v$ or $< -v$.

For $v = 12''$ this becomes

$$\frac{k^2}{h^2} \cdot \left(\frac{1}{2} + h^2 \frac{v_1^2 + v_2^2}{2}\right) = \frac{105}{3735} \times 12^2 = 4\cdot05.$$

The value of $\frac{k}{h}$ is obtained from the cross motions and $= 1.''44$.

Thus

$$h^2 \frac{v_1^2 + v_2^2}{2} = 1\cdot46 \quad . \quad . \quad . \quad (10)$$

Again

$$k\sqrt{\pi} \cdot \frac{v_1 + v_2}{2} = \text{parallactic motion.}$$

With the value of $\frac{k}{h}$ given above, this equation becomes

$$hv_1 + hv_2 = 1\cdot13 \quad . \quad . \quad . \quad (11)$$

Solution of these equations gives $hv_1 = 1\cdot65$ and $hv_2 = -\cdot52$. These values give the proportion of large positive to large negative proper motions satisfactorily. For the small proper motions they give two maxima at $+1''\cdot7$ and $-0''\cdot5$ approximately. The existence of two maxima finds a little support in the observations, but the small proper motions are not spread out so much as the hypothesis of two equal streams requires. The distribution given by two equal streams with $hv_1 = 1\cdot65$ and $hv_2 = -\cdot52$ is shown by C in the diagram.

9. I have modified equations (10) and (11) on the supposition that the number of stars in the two streams was different. This does not, however, remove the difficulty. With the small value $1\cdot44$ of $\frac{k}{h}$, the curve corresponding to A of the diagram is very much contracted. Large values of hv_1 and hv_2 are therefore necessary to represent the numbers of large positive and negative proper motions. A difference in the numbers belonging to the two streams changes the height of the two maxima, but does not lessen their distance apart. Consequently too few stars of small proper motion are given on this supposition. This could be remedied by the introduction of a third stream with small positive value of hv, similar to the one found by Dr. Halm in his investigations.

A Comparison of the Moon's Co-ordinates for 1914 *according to the New Delaunay Tables with those given in the "Nautical Almanac."* By A. C. D. Crommelin, D.Sc.

(*Communicated by the Astronomer Royal.*)

The errors of the *Nautical Almanac* co-ordinates of the Moon have now become so large that a comparison with the new Delaunay Tables is of special interest and importance. The material for a comparison is afforded by the table given in the *Connaissance des Temps* for 1914, pp. 667–669. This gives the values of "Hansen *minus* Delaunay" for longitude, latitude, and parallax for each noon of 1914. The addition of the quantities in this table to those in the tables in the Appendix to the *Nautical Almanac* for 1914, pp. 12, 13, 16, and the subsequent change of sign of the sums gives us the quantities in Table I., which are the values of "Delaunay *minus* Hansen-Newcomb" for longitude, latitude, and parallax. Table II. gives the corresponding values of "Delaunay *minus* Hansen-Newcomb" for right ascension and declination, or, in other words, the corrections required to the right ascension and declination of the Moon as given in the *Nautical Almanac*, on the assumption that the new Tables are accurate.

The following are the mean values of "Delaunay *minus* Hansen-Newcomb" for the different elements in the year 1914 :—

Longitude.	Latitude.	Right Ascension.	Parallax.	Semi-diameter.
		s		
+9″·7	+1″·1	+·664	+″·47	−″·49

The constant of parallax used in the Delaunay Tables is practically identical with that found gravitationally by Newcomb, his value being 57′ 2″·68, while Hansen's was 57′ 2″·23. The residual of 1″·1 in latitude is due to the omission in the new Tables of the constant term which Hansen inserted, owing to his conclusion that the centre of gravity of the Moon did not coincide with its centre of figure.

The inclination of the Moon's orbit is increased $3\frac{1}{3}''$, from 5° 8′ 39″·96 to 43″·3; the longitude of ascending node, 1900, Jan. 1·0, Paris M.T., is increased 11″·8, from 259° 7′ 40″·97 to $52''\cdot79$. The centennial retrogradation is diminished by $11\frac{1}{4}''$.

It will be seen that in addition to the alteration in the mean parallax, there are sensible oscillations of short period, which arise from small alterations in the eccentricity, evection, variation, etc. The eccentricity is altered from ·05490857 to ·05490056, which is equivalent to the introduction of a term in the parallax with coefficient 0″·025 and period an anomalistic month. The coefficient of the evection term in the parallax appears to be diminished by about 0″·03.

Table II. has been calculated from Table I. for the times of

the Moon passing 0^h, 3^h, 6^h, 9^h . . . 21^h. Some abbreviations have been used for the intermediate dates, which may occasionally cause the quantities in Table II. to deviate $^s\cdot02$ or $0''\cdot3$ from those corresponding rigorously to Table I.

A Table has been constructed at Greenwich for correction of the Hansen-Newcomb longitudes in recent years ;* in this the correction of mean longitude was made $+9''\cdot6$ at 1913·0, $+10''\cdot0$ at 1914·0; the term $+1''\cdot56 \sin D$ was used for correcting the parallactic inequality, and $-0''\cdot55 \sin g$ for correcting the eccentricity ; the following Jupiter and Venus terms were also used :—

$$-1''\cdot1 \sin(2D - g + 2E - 2J) \; -0''\cdot68 \sin(2D - g + 3V - 3E)$$
$$+0''\cdot32 \sin(g + 2\varpi - 3J + 7^\circ) \; -0''\cdot35 \sin(2V - 3E + 85^\circ).$$

This Table makes the corrections to the *Nautical Almanac* right ascension and declination 1914 Jan. 1, $+^s\cdot64$, $+4''\cdot8$, practically identical with the values in Table II. As the Greenwich correction to right ascension has been too small in 1912 and the beginning of 1913 by about $0^s\cdot10$ in the mean, it is likely that the correction to right ascension in Table II. will also be somewhat too small in the mean.

It is also of interest to compare the quantities in Table I. with the supplementary corrections (also due to Newcomb) used in the *American Ephemeris* for the eclipses of 1914 ; these are denoted by *A.E.* in the following Table :—

G.M.T. 1914.			*A.E.* – Hansen-Newcomb.			Delaunay – *A.E.*		
	d	h	Long.	Lat.	Par.	Long.	Lat.	Par.
			$''$	$''$	$''$	$''$	$''$	$''$
Feb.	24	11	+8·4	−0·1	+0·4	+0·6	+1·4	+0·1
Mar.	11	17	+8·3	+0·5	+0·3	+1·7	+2·0	+0·1
Aug.	21	0	+8·8	+0·5	+0·4	+2·2	+0·9	+0·1
Sept.	4	2	+8·9	−0·2	+0·4	+0·4	+0·6	+0·1

It will be seen that the *A.E.* corrections are probably too small, and that the *A.E.* central line of the total eclipse of August 21 should be shifted through $\frac{1}{4}$ of its distance from the *N.A.* central line, in the direction away from the latter.

Perhaps it may not be out of place to notice here an interesting numerical relationship. According to the Delaunay Tables, 100 lunations are equal to 2953·05881 days. Now the fraction $\frac{1}{17} = 0\cdot058824$.

That is, 1700 lunations are exactly commensurate with the solar day, to the degree of accuracy with which the Moon's motion is known. They are equal to $17 \times 2953 + 1$ day, or 50202 days ; hence 850 lunations = 25101 days (about 68·72 years). Also, as the ephemeris is computed at intervals of half a day, the variation and parallactic inequality recur exactly after $12550\frac{1}{2}$ days, this being equal to 425 lunations.

* The object of this table was to give improved tabular places for comparison with the observations, and also for the use of certain Boundary Commissioners.

Table I.

Corrections to the Hansen-Newcomb Longitudes, Latitudes, and Parallaxes of the Moon for 1914, *according to the new Delaunay Tables.*

Noon.	January.			February.			March.			April.		
	Long.	Lat.	Par.	Long.	Lat.	Par.	Long.	Lat.	Par.	Long.	Lat.	Par.
	″	″	″	″	″	″	″	″	″	″	″	″
1	+10·2	+1·0	+·37	+10·9	+0·1	+·32	+ 9·8	−0·2	+·37	+11·3	−0·3	+·34
2	10·5	+0·8	·35	11·1	+0·3	·31	10·3	−0·5	·34	11·9	−0·4	·23
3	10·8	+0·6	·33	11·1	+0·4	·29	10·8	−0·6	·29	12·4	−0·2	·18
4	11·0	+0·3	·31	11·1	+0·2	·30	11·0	−0·7	·26	12·7	+0·2	·25
5	11·0	+0·2	·29	11·2	+0·1	·32	11·0	−0·5	·25	12·7	+0·7	·40
6	11·0	+0·1	·27	11·3	+0·2	·34	11·1	0·0	·30	12·2	+1·0	·58
7	10·9	0·0	·27	11·3	+0·4	·35	11·3	+0·5	·40	11·6	+1·2	·66
8	11·0	0·0	·26	11·1	+0·7	·35	11·3	+0·9	·49	11·0	+1·3	·62
9	10·6	+0·1	·26	10·6	+1·2	·33	11·0	+1·2	·54	10·6	+1·6	·49
10	10·0	+0·5	·27	10·1	+1·8	·35	10·7	+1·6	·50	10·0	+2·1	·33
11	9·4	+0·9	·29	9·5	+2·4	·42	10·4	+2·2	·41	9·3	+2·4	·26
12	8·9	+1·5	·34	9·1	+2·5	·53	9·9	+2·6	·37	8·5	+2·4	·31
13	8·8	+2·1	·41	8·8	+2·2	·69	9·2	+2·8	·40	8·0	+2·2	·44
14	8·8	+2·3	·50	8·6	+1·7	·78	8·4	+2·6	·56	7·8	+1·9	·54
15	8·8	+2·2	·55	8·2	+1·7	·77	7·9	+2·0	·69	7·8	+1·8	·59
16	8·7	+1·9	·54	7·9	+1·9	·68	7·8	+1·9	·71	7·7	+1·7	·61
17	8·4	+1·8	·52	7·7	+2·2	·57	7·8	+1·9	·70	7·6	+1·6	·61
18	8·3	+1·8	·48	7·6	+2·5	·51	7·7	+2·2	·64	7·4	+1·4	·60
19	8·3	+2·0	·45	7·6	+2·7	·50	7·4	+2·3	·60	7·5	+1·2	·57
20	8·5	+2·1	·45	7·8	+2·6	·51	7·2	+2·2	·58	7·7	+1·0	·57
21	8·7	+2·1	·47	7·9	+2·4	·52	7·3	+2·0	·57	8·1	+0·9	·58
22	8·9	+2·1	·48	8·2	+2·1	·53	7·6	+1·8	·56	8·6	+0·7	·60
23	9·0	+2·1	·49	8·5	+1·9	·53	8·0	+1·5	·56	9·1	+0·6	·60
24	9·3	+2·1	·50	8·8	+1·5	·52	8·4	+1·3	·57	9·6	+0·6	·59
25	9·4	+2·0	·50	9·3	+1·1	·50	8·8	+1·1	·56	10·1	+0·7	·57
26	9·6	+1·7	·48	9·5	+0·7	·46	9·3	+0·8	·55	10·2	+0·8	·53
27	9·7	+1·4	·45	9·6	+0·3	·42	9·7	+0·5	·53	10·1	+0·7	·49
28	9·9	+1·2	·43	9·7	0·0	·39	9·7	+0·4	·49	10·2	+0·7	·46
29	10·1	+0·8	·39				9·6	+0·1	·45	10·4	+0·5	·44
30	10·3	+0·4	·36				10·0	0·0	·41	11·1	+0·2	·39
31	10·5	+0·1	·34				10·6	−0·2	·40			

TABLE I.—*continued.*

Noon.	May.			June.			July.			August.		
	Long.	Lat.	Par.	Long.	Lat.	Par.	Long.	Lat.	Par.	Long.	Lat.	Par.
	"	"	"	"	"	"	"	"	"	"	"	"
1	+11·9	+0·1	+·30	+12·2	+0·6	+·30	+12·6	+1·6	+·31	+12·7	+2·3	+·40
2	12·7	+0·2	·24	12·8	+0·9	·34	12·8	+1·9	·40	12·2	+2·2	·46
3	13·4	+0·4	·25	12·9	+1·2	·43	12·7	+1·9	·49	11·5	+1·9	·49
4	13·6	+0·7	·35	12·4	+1·1	·52	12·2	+2·0	·52	10·8	+1·6	·50
5	13·2	+0·9	·50	11·8	+1·1	·56	11·6	+2·0	·52	10·3	+1·4	·51
6	12·4	+0·9	·63	11·0	+1·2	·51	11·0	+2·0	·52	9·7	+1·1	·54
7	11·4	+0·8	·59	10·4	+1·5	·45	10·5	+2·0	·51	9·3	+0·7	·59
8	10·8	+0·8	·45	9·9	+1·7	·42	10·0	+1·7	·53	8·8	+0·3	·62
9	10·1	+1·3	·34	9·3	+1·7	·43	9·5	+1·4	·57	8·4	+0·1	·65
10	9·4	+1·7	·29	8·7	+1·4	·50	8·9	+1·0	·59	8·1	−0·1	·62
11	8·7	+1·9	·33	8·1	+1·2	·55	8·3	+0·6	·58	7·9	−0·2	·57
12	8·0	+1·8	·41	7·8	+1·0	·59	8·2	+0·3	·57	7·8	−0·2	·51
13	7·7	+1·5	·49	7·6	+0·8	·57	8·2	+0·2	·54	7·9	−0·1	·46
14	7·5	+1·3	·52	7·6	+0·8	·56	8·2	0·0	·51	7·8	0·0	·44
15	7·4	+1·3	·54	7·7	+0·7	·53	8·2	−0·1	·48	8·1	+0·2	·43
16	7·4	+1·2	·54	7·8	+0·7	·51	8·5	−0·2	·48	8·8	+0·3	·46
17	7·3	+1·1	·54	7·9	+0·7	·51	9·0	−0·1	·50	9·5	+0·4	·50
18	7·3	+1·1	·55	8·2	+0·7	·54	9·7	0·0	·52	10·0	+0·5	·51
19	7·5	+1·0	·56	8·8	+0·5	·60	10·2	+0·2	·58	10·2	+0·8	·52
20	7·8	+0·9	·59	9·4	+0·5	·63	10·6	+0·3	·60	10 5	+1·1	·52
21	8·3	+0·9	·61	9·8	+0·6	·63	10·7	+0·5	·61	11·0	+1·4	·51
22	8·9	+0·9	·62	10·1	+0·6	·61	10·9	+0·8	·60	11·5	+1·6	·52
23	9·5	+0·9	·61	10·5	+0·7	·57	11·2	+1·0	·56	11·6	+1·8	·52
24	9·9	+0·9	·59	10·7	+0·6	·53	11·6	+1·1	·51	11·3	+1·9	·48
25	10·0	+0·9	·55	10·8	+0·6	·50	11·8	+1·2	·45	10·8	+2·1	·37
26	10·1	+0·8	·51	11·0	+0·7	·48	11·7	+1·4	·40	10·4	+2·4	·23
27	10·0	+0·8	·49	11·0	+0·9	·45	11·6	+1·7	·32	10·5	+2·7	·14
28	10·1	+0·7	·48	11·1	+1·0	·40	11·7	+2·0	·26	10·9	+2·7	·15
29	10·2	+0·7	·44	11·5	+1·1	·35	12·2	+2·3	·23	11·4	+2·3	·25
30	10·7	+0·5	·40	12·2	+1·3	·30	12·6	+2·5	·25	11·7	+1·8	·39
31	11·4	+0·4	·34				12·9	+2·5	·32	11·5	+1·5	·50

TABLE. I.—*continued.*

Noon.	September.			October.			November.			December.		
	Long.	Lat.	Par.	Long.	Lat.	Par.	Long.	Lat.	Par.	Long.	Lat.	Par.
	″	″	″	″	″	″	″	″	″	″	″	″
1	+11·0	+1·3	+·53	+10·3	+0·9	+·55	+9·5	+0·7	+·48	+8·9	+0·6	+·44
2	10·5	+0·7	·53	10·0	+0·8	·54	9·2	+0·6	·48	8·6	+0·7	·45
3	9·9	+0·6	·54	9·6	+0·7	·54	9·1	+0·5	·49	8·5	+0·7	·48
4	9·3	+0·5	·56	9·3	+0·5	·53	9·0	+0·4	·51	8·4	+1·0	·50
5	8·9	+0·3	·57	8·9	+0·5	·54	9·0	+0·3	·52	8·5	+1·1	·51
6	8·4	+0·1	·59	8·7	+0·3	·54	8·9	+0·3	·53	8·4	+1·2	·52
7	8·0	0·0	·59	8·5	+0·3	·54	9·0	+0·4	·53	8·5	+1·3	·51
8	7·8	+0·1	·59	8·3	+0·2	·52	9·0	+0·4	·54	8·7	+1·2	·50
9	7·6	+0·2	·56	8·3	+0·3	·50	9·0	+0·5	·54	8 8	+1·3	·47
10	7·5	+0·3	·50	8·2	+0·5	·49	9·0	+0·7	·55	8·9	+1·5	·52
11	7·4	+0·5	·47	8·1	+0·6	·50	9·2	+0·8	·60	9·0	+1·6	·59
12	7·2	+0·6	·45	8·2	+0·7	·52	9·4	+0·8	·67	9·1	+1·3	·64
13	7·5	+0·8	·45	8·6	+0·6	·55	9·9	+0·7	·70	9·6	+1·0	·69
14	8·1	+0·8	·45	9·2	+0·5	·58	10·5	+0·6	·66	10·3	+1·2	·62
15	8·8	+0·6	·46	9·9	+0·3	·56	11·0	+0·9	·54	10·8	+1·7	·45
16	9·6	+0·6	·45	10·4	+0·2	·49	11·2	+1·4	·39	11·1	+2·2	·30
17	10·0	+0·6	·43	10·7	+0·5	·40	11·2	+2·0	·29	11·1	+2·3	·27
18	10·4	+0·8	·40	11 0	+1·0	·32	10·8	+2·2	·33	10·7	+2·2	·33
19	10·8	+1·1	·42	11·2	+1·4	·35	10·3	+2·1	·45	10·2	+1·9	·40
20	11·3	+1·4	·45	11·2	+1·6	·49	9·9	+2·0	·57	9·9	+1·5	·45
21	11·6	+1·5	·54	11·0	+1·7	·60	9·5	+2·0	·53	9·8	+1·3	·44
22	11·3	+1·7	·58	10·5	+2·0	·61	9·2	+2·2	·47	9·8	+1·0	·42
23	10·6	+2·0	·52	9·8	+2·6	·51	9·1	+2·0	·38	9·9	+0·8	·38
24	9·7	+2·6	·37	9·1	+3·0	·35	9·0	+1·6	·35	10·0	+0·6	·36
25	9·1	+3·0	·21	8·8	+2·9	·24	9·0	+1·1	·35	10·0	+0·3	·35
26	9·1	+3·0	·14	8·7	+2·4	·23	9·2	+0·7	·38	10·0	+0·2	·35
27	9·6	+2·5	·20	9·1	+1·7	·32	9·5	+0·5	·41	10·3	+0·1	·36
28	10·2	+1·8	·35	9·6	+1·1	·43	9·8	+0·5	·43	10·3	+0·2	·37
29	10·5	+1·2	·48	10·0	+0·9	·49	9·7	+0·5	·44	10·1	+0·3	·38
30	10·6	+1·0	·54	10·0	+0·9	·52	9·4	+0·6	·44	9·8	+0·5	·39
31				9·7	+0·9	·50				9·3	+0·7	·41
32										9·0	+1·1	·43

Table II.

Corrections to the "Nautical Almanac" Right Ascensions and Declinations of the Moon in 1914 according to the Delaunay Tables.

Noon.	January.		February.		March.		April.		May.		June.	
	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.
	s	"	s	"	s	"	s	"	s	"	s	"
1	+·62	+4·7	+·66	+4·4	+·64	+3·2	+·84	+1·1	+·87	−1·2	+·78	−3·8
2	+·62	+4·9	+·70	+4·1	+·69	+2·7	+·90	0·0	+·89	−2·4	+·80	−4·2
3	+·64	+4·8	+·74	+3·6	+·75	+2·2	+·93	−1·0	+·90	−3·5	+·81	−4·3
4	+·67	+4·6	+·78	+2·9	+·80	+1·5	+·93	−1·9	+·88	−4·3	+·80	−4·0
5	+·69	+4·2	+·81	+2·1	+·83	+0·8	+·90	−2·6	+·84	−4·4	+·80	−3·1
6	+·72	+3·8	+·84	+1·1	+·84	0·0	+·83	−2·9	+·78	−4·2	+·79	−1·9
7	+·75	+3·2	+·85	0·0	+·85	−0·8	+·76	−3·2	+·74	−3·6	+·78	−0·5
8	+·78	+2·4	+·83	−0·9	+·83	−1·4	+·70	−3·1	+·72	−2·8	+·75	+0·8
9	+·79	+1·5	+·77	−1·4	+·78	−1·8	+·67	−2·8	+·70	−1·9	+·70	+1·9
10	+·76	+0·7	+·70	−1·7	+·73	−2·1	+·66	−2·0	+·68	−0·8	+·63	+2·7
11	+·71	+0·1	+·64	−1·8	+·70	−1·9	+·65	−1·1	+·65	+0·4	+·56	+3·1
12	+·66	−0·2	+·61	−1·7	+·67	−1·6	+·63	−0·1	+·61	+1·5	+·51	+3·3
13	+·64	−0·4	+·60	−1·5	+·63	−1·4	+·61	+0·6	+·57	+2·3	+·47	+3·4
14	+·63	−0·8	+·59	−1·3	+·58	−1·0	+·59	+1·3	+·53	+2·8	+·45	+3·6
15	+·61	−1·3	+·58	−1·0	+·57	−0·6	+·58	+1·9	+·49	+3·2	+·46	+3·8
16	+·58	−1·7	+·58	−0·5	+·59	0·0	+·55	+2·5	+·46	+3·5	+·46	+3·8
17	+·57	−1·6	+·58	+0·4	+·59	+0·8	+·52	+3·0	+·44	+3·7	+·47	+3·7
18	+·57	−1·4	+·58	+1·5	+·59	+1·7	+·48	+3·4	+·42	+3·8	+·51	+3·5
19	+·58	−1·1	+·58	+2·4	+·56	+2·4	+·47	+3·7	+·43	+3·9	+·58	+3·3
20	+·63	−0·5	+·57	+3·2	+·51	+2·9	+·46	+3·9	+·45	+3·9	+·64	+3·0
21	+·67	+0·3	+·55	+3·7	+·49	+3·4	+·48	+4·0	+·49	+4·0	+·70	+2·5
22	+·68	+1·1	+·53	+4·1	+·47	+3·8	+·51	+4·1	+·55	+4·0	+·75	+1·7
23	+·69	+2·0	+·53	+4·4	+·48	+4·0	+·54	+4·2	+·62	+3·9	+·79	+0·7
24	+·68	+3·0	+·53	+4·5	+·50	+4·2	+·58	+4·1	+·68	+3·4	+·80	−0·4
25	+·65	+3·7	+·54	+4·5	+·53	+4·3	+·62	+3·9	+·73	+2·6	+·79	−1·4
26	+·63	+4·2	+·56	+4·4	+·55	+4·2	+·67	+3·7	+·76	+1·6	+·77	−2·2
27	+·60	+4·5	+·58	+4·1	+·57	+4·0	+·71	+3·1	+·76	+0·6	+·74	−2·9
28	+·60	+4·7	+·60	+3·7	+·60	+3·7	+·75	+2·2	+·76	−0·5	+·72	−3·4
29	+·60	+4·5			+·64	+3·3	+·79	+1·1	+·74	−1·5	+·74	−3·6
30	+·61	+4·5			+·69	+2·8	+·83	0·0	+·73	−2·5	+·78	−3·8
31	+·63	+4·5			+·77	+2·1			+·75	−3·3		

TABLE II.—*continued.*

Noon.	July.		August.		September.		October.		November.		December.	
	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.	R.A.	Dec.
	s	″	s	″	s	″	s	″	s	″	s	″
1	+·82	−3·6	+·96	+0·7	+·75	+3·7	+·62	+4·5	+·57	+4·2	+·62	+2·8
2	+·86	−3·0	+·93	+1·8	+·68	+4·0	+·59	+4·4	+·58	+3·7	+·63	+2·2
3	+·89	−2·0	+·87	+2·8	+·61	+3·9	+·57	+4·2	+·60	+3·1	+·63	+1·5
4	+·90	−0·7	+·77	+3·6	+·56	+3·8	+·56	+4·0	+·62	+2·5	+·63	+0·8
5	+·87	+0·6	+·69	+4·0	+·53	+3·6	+·55	+3·7	+·64	+1·9	+·63	+0·2
6	+·83	+1·9	+·62	+4·1	+·51	+3·4	+·56	+3·3	+·66	+1·1	+·62	−0·4
7	+·76	+2·9	+·58	+3·9	+·49	+3·1	+·58	+2·7	+·67	+0·1	+·61	−1·0
8	+·69	+3·5	+·54	+3·7	+·50	+2·9	+·60	+2·1	+·67	−0·8	+·60	−1·5
9	+·62	+3·9	+·51	+3·5	+·50	+2·7	+·62	+1·5	+·65	−1·4	+·59	−1·9
10	+·56	+3·9	+·50	+3·2	+·51	+2·5	+·63	+0·8	+·63	−2·0	+·59	−2·0
11	+·50	+3·8	+·49	+3·0	+·53	+2·2	+·62	+0·1	+·61	−2·6	+·59	−2·1
12	+·48	+3·6	+·51	+2·7	+·55	+1·7	+·60	−0·7	+·61	−3·0	+·61	−2·1
13	+·50	+3·4	+·53	+2·4	+·57	+1·0	+·61	−1·5	+·63	−3·4	+·65	−2·0
14	+·51	+3·2	+·55	+2·0	+·60	0·0	+·63	−2·3	+·66	−3·6	+·72	−1·8
15	+·53	+2·9	+·59	+1·6	+·64	−1·0	+·64	−3·1	+·71	−3·2	+·79	−1·2
16	+·57	+2·7	+·66	+1·0	+·67	−2·0	+·65	−3·6	+·76	−2·3	+·82	+0·1
17	+·62	+2·5	+·73	+0·2	+·66	−2·7	+·66	−3·7	+·82	−1·0	+·82	+1·7
18	+·69	+2·3	+·74	−0·6	+·65	−3·0	+·69	−3·5	+·83	+0·3	+·78	+2·9
19	+·76	+1·7	+·73	−1·3	+·68	−3·1	+·74	−3·0	+·80	+1·6	+·70	+3·8
20	+·80	+0·8	+·73	−2·0	+·72	−3·1	+·80	−2·1	+·75	+2·7	+·64	+4·3
21	+·81	−0·2	+·74	−2·6	+·76	−2·9	+·82	−0·8	+·68	+3·6	+·61	+4·5
22	+·81	−1·0	+·75	−3·1	+·78	−2·2	+·81	+0·6	+·61	+4·4	+·59	+4·6
23	+·81	−1·8	+·76	−3·1	+·77	−1·0	+·77	+2·0	+·56	+4·8	+·59	+4·6
24	+·79	−2·5	+·75	−2·6	+·74	+0·5	+·69	+3·1	+·53	+4·7	+·60	+4·5
25	+·77	−3·0	+·75	−2·0	+·70	+2·1	+·62	+3·9	+·52	+4·5	+·62	+4·3
26	+·76	−3·1	+·75	−1·2	+·69	+3·1	+·56	+4·5	+·53	+4·3	+·64	+3·9
27	+·76	−3·0	+·79	−0·3	+·70	+3·9	+·56	+4·9	+·56	+4·1	+·67	+3·5
28	+·79	−2·7	+·84	+0·5	+·70	+4·2	+·56	+5·2	+·59	+3·9	+·70	+3·0
29	+·83	−2·4	+·88	+1·4	+·69	+4·3	+·56	+5·2	+·61	+3·7	+·72	+2·4
30	+·90	−1·7	+·88	+2·5	+·67	+4·4	+·56	+4·9	+·62	+3·4	+·72	+1·7
31	+·95	−0·6	+·83	+3·3			+·57	+4·5			+·70	+1·0
32											+·67	+0·2

On a Suggested Substitute for Bode's Law. By M. A. Blagg. (Plate 12.)

(*Communicated by Professor H. H. Turner.*)

I must begin by gratefully acknowledging my indebtedness to Professor Turner for the help he has given me in drawing up this paper, but for which it would have been very crude indeed in many respects.

1. It is a familiar fact that Bode's Law is only approximately fulfilled. Thus the late Professor C. A. Young writes in his *General Astronomy*, Art. 488.

"The resulting numbers, divided by 10, are pretty nearly the true mean distances of the planets from the sun, in terms of the radius of the earth's orbit. In the case of Neptune, however, the law breaks down utterly, and is not even approximately correct."

Professor Young gives a table in support of this statement, from which the first three columns of the subjoined table are quoted. It will be seen that his special condemnation of the law for Neptune arises from his exhibition of the simple *differences* between the actual distances and those given by Bode. But if we take instead the *ratios*, Neptune becomes less conspicuous; though still notably discordant. It is not three times as discordant as Venus. Moreover, Mercury is only brought into approximate accordance by tampering with the geometrical progression. The term preceding 0·7 in Bode's Law is really 0·55, and if we use this, then the ratio is no longer ·968 but ·704, even less than for Neptune.

TABLE I.

From Professor Young's Table.

Name.	Distance.	Bode.	Difference.	Ratio.	Residual
Mercury . .	0·387	0·4	−0·013	·968	+0·012
Venus . .	0·723	0·7	+0·023	1·033	+0·077
Earth . .	1·000	1·0	0·000	1·000	+0·044
Mars . . .	1·523	1·6	−0·077	·952	−0·004
Mean Asteroids .	2·650	2·8	−0·150	·947	−0·009
Jupiter . .	5·202	5·2	+0·002	1·000	+0·044
Saturn . .	9·539	10·0	−0·461	·954	−0·002
Uranus . .	19·183	19·6	−0·417	·978	+0·022
Neptune . .	30·054	38·8	−8·746!	·774	−0·182
Mean	...	...	...	·956	...

To obviate this defect Professor C. V. L. Charlier has recently

suggested (*Astr. Nachr.*, 4623) that Mercury really corresponds in the series

$$d = 0{\cdot}4 + 0{\cdot}3 \times 2^n$$

(in which $n = 1$ for the earth, and $n = 0$ for Venus) *not* to the value $n = -1$, but to the value $n = -\infty$; so that in between Venus and Mercury we may look for a large number of small planets with values of n between 0 and $-\infty$: corresponding perhaps to Saturn's ring.

2. It occurred to me, towards the end of last year, to look at the matter from a different point of view. Bode's Law is a modified geometrical series, the constant 0·4 being added to remove the chief deviations from a strictly geometrical series, to which the law tends when n is large. Let us omit this modification and return to the notion of a geometrical series, at least in the first instance. If we take logarithms of the distances (as in the 2nd column of Table II.) the differences ought to be constant.

Table II.

Planet.	Distance.	Log.	Difference.	Theoret.	O−C.	Number.
Mercury . .	0·3871	9·588		9·551	0·037	1·089
			·271			
Venus . .	0·7233	9·859		9·789	0·070	1·175
			·141			
Earth . .	1·0000	0·000		0·026	9·974	0·942
			·183			
Mars . .	1·5237	0·183		0·264	9·919	0·830
			·240			
(Min. P.) .	(2·650)	(0·423)		(0·501)	9·922	0·836
			·293			
Jupiter . .	5·2026	0·716		0·739	9·977	0·948
			·264			
Saturn . .	9·5547	0·980		0·976	0·004	1·009
			·304			
Uranus . .	19·2181	1·284		1·214	0·070	1·175
			·195			
Neptune .	30·1096	1·479		1·451	0·028	1·067
Mean	...	0·501	·236	...	...	...

Note.—The 3rd column shows incidentally the flaws in Bode's Law, which demands that the differences shall increase continuously, approaching more and more nearly to ·301 as n grows larger. The values for "Distance" are taken from the *Annuaire du Bureau des Longitudes.*

From the 3rd column we see that the range is considerable, the departures from the mean value ·236 amounting to 30 or 40 per cent. Let us, nevertheless, construct a perfectly regular series as in the 4th column, starting with the mean value* of the log (distances), viz. 0·501, for the fifth term, and adding or subtracting a constant each time.

3. But instead of the mean value 0·236 we shall adopt the slightly larger constant difference $0{\cdot}23742 = \log 1{\cdot}7275$, a number

* Note that the starting-point is arbitrary. This point becomes important later.

to which I was gradually brought by much "trial and error," and which will be justified immediately. We thus get the 4th column, and can compare it with the 2nd, under the heading O – C. Now, it is seen at once that these numbers form a smooth curve, rather like a variable star curve. Venus and Uranus provide the maxima, or nearly so; and the number 1·7275, or its logarithm ·23742, was chosen so as to make the maxima of the same height as nearly as possible. We cannot change these numbers much without destroying this equality; but it is just possible that the true ratio may be a very little greater than 1·7275—I am sure it is not less. The single minimum is near Mars and the minor planets.

4. Having got so far, and having interested myself for a time by the harmonic analysis of the numbers shown in the 6th column of Table II., I showed the results to Professor Turner; and feeling encouraged by the interest he expressed in them, I next turned to the satellites of Jupiter. To my great surprise I found (almost in spite of myself, for I started with a very different ratio suggested by the earlier satellities) the *same mean ratio*, 1·7275.

TABLE III.

Satellites of Jupiter.

Satellite.	Distance.	Log.	Diff.	Theoret.	O–C.	Number.	O-C Corrected.	Number.
V.	0·4297	9·633		9·540	0·093	1·239	0·011	1·026
			·183					
	...	...		9·778	...	...	...	...
			·184					
I.	1·000	0·000		0·015	9·985	0·966	9·903	·800
			·202					
II.	1·5917	0·202		0·252	9·950	0·891	9·868	·738
			·203					
III.	2·5390	0·405		0·490	9·915	0·822	9·833	·681
			·245					
IV.	4·4657	0·650		0·727	9·923	0·837	9·841	·693
			·263					
	...	...		0·965	...	...	...	...
			·263					
	...	...		1·202	...	...	...	..
			·263					
VI. and VII.	(27·5)	1·439		1·440	9·999	0·998	9·917	·826
			·305					
VIII.	55·437	1·744		1·677	0·067	1·167	9·985	·966
Mean	...	...	·235	...	...	..	...	...

The effect of using this ratio is shown in Table III., on the same plan as Table II. Satellites VI. and VII. are taken together, the distance 27·5 being about midway between their individual distances.

5. It will be seen from the 5th and 6th columns that we have got a curve of the same kind as before, the only differences being—

(1) The successive satellites follow one another more closely on

the curve, for whereas in Table II. the interval between two maxima was about six steps (Venus to Uranus), in Table III. we barely cover the interval in nine steps. There is no question that the step of Table III. is different from that of Table II. Whether it bears a simple relation to it (such as $\frac{2}{3}$) we need not consider just yet.

(2) Again, the "numbers" in the last column of Table II. range from 1·175 to 0·830, giving a difference of 0·345 (This is not quite the same as the range from maximum to minimum of the *curve* through the points; but for a moment let us consider it the same.) The corresponding "numbers" in column 6 of Table III. range from 1·239 to 0·822, a difference of 0·417; and hence the ordinates of the curves will not fit as they stand. But to make the ranges equal we have only to alter the "Theoretical" logarithms by a constant, viz. the constant

$$\log 0{\cdot}417 - \log 0{\cdot}345 = 0{\cdot}082.$$

If we add this to the 4th column (Theoretical), then it will be subtracted from the next column. The results of this subtraction are given in column 7, and the corresponding numbers are given in column 8, where the range is now 1·026 − ·681 = 0·345 for Table III. as against 1·175 − ·830 = 0·345 for Table II. As already remarked, these are not actually the ranges which we wish to make coincident; but the ranges of the curves can clearly be adjusted in the same way.

6. Now this process is quite legitimate, because the zero or starting-point for the "Theoretical" logarithms is arbitrary. In Table II. we chose for it the mean of the 9 planets: but there is even then a question whether we should have included the quite arbitrary value for the minor planets. In Table III. the arbitrary nature of the choice is brought out more clearly by the more numerous gaps. There is really nothing as yet to guide us. It may be that we shall find some selected point which ought to correspond on the two curves, but consideration of the point must be reserved for the present. The fact that the choice is arbitrary is important in proceeding to the case of Saturn's system which was at first puzzling, and might not have been brought into line if the agreement of the Sun's and Jupiter's had not suggested keeping

(*a*) The same ratio 1·7275;

(*b*) The same curve of deviation.

7. We are thus faced with the question, Can Saturn's system be made to fit this ratio and curve by a suitable choice of the arbitrary constant in the column of "Theoretical" log distances? My answer is in the affirmative, as shown in Table IV. It will be seen at once that there are two new departures. First, the differences in the 4th column are in most cases sensibly less than 0·2374, and it would not, without the precedent of the Sun and Jupiter, have suggested itself to use this value as before. We

may remark, however, that the corresponding early ratios for Jupiter's system are distinctly small: so that Saturn's system is not without a parallel in this respect. Secondly, the "Numbers" in the final column are no longer all near unity; they are very much smaller. But they fit on the curve. They do not rise to a maximum, and they are very close together, the steps being now about $\frac{1}{5}$ or $\frac{1}{6}$ of those in Table II. But, in addition to the fit which has thus been obtained, there is another reason (connected with Saturn's ring), to be mentioned later, which supports the view that the system of Saturn is properly represented.

TABLE IV.

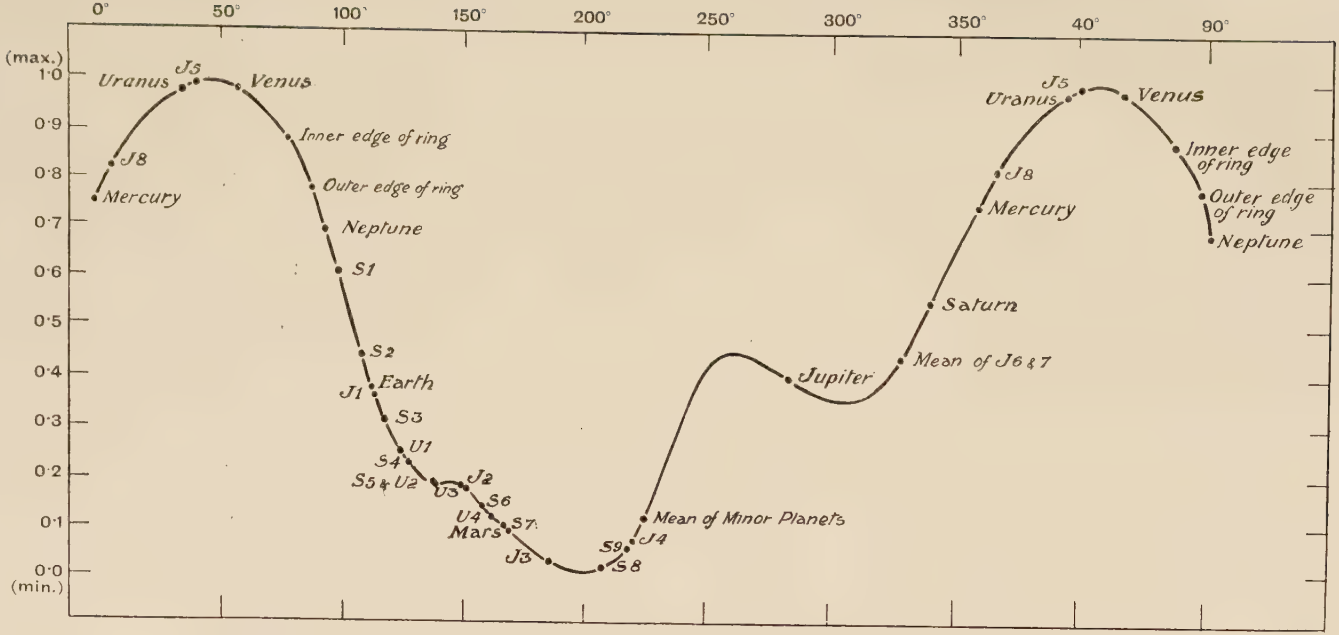
Satellites of Saturn.

Satellites.	Distance.	Log.	Difference.	Theoret.	O–C.	Number.
1	·629	9·799		0·410	9·389	0·245
			0·108			
2	·808	9·907		0·648	9·259	0·182
			0·093			
3	1·000	0·000		0·885	9·115	0·130
			0·108			
4	1·280	0·108		1·122	8·986	0·097
			0·144			
5	1·787	0·252		1·360	8·892	0·078
			0·183			
...	...	...		1·597	...	...
			0·183			
6	4·147	0·618		1·835	8·783	0·061
			0 083			
7	5·027	0·701		2·072	8·629	0·043
			0·127			
...	...	...		2·310	...	...
			0·127			
...	...	...		2·547	...	...
			0·127			
8	12·09	1·082		2·784	8·298	0·020
			0·280			
...	...	...		3·022	...	...
			0·279			
9	43·78	1·641		3·259	8·382	0·024
Mean	...	...	0·154	...	...	...

8. Let us now represent the curve shown in the diagram by $f(\theta)$ where θ is given values of the form $\alpha + n\beta$; α and β being two constants, and n representing successive integers. Then the distance of a satellite or a planet is represented by the formula

$$\text{Distance} = A(1{\cdot}7275)^n\{B + f(\alpha + n\beta)\}$$

There are thus four constants, A, B, α, β to be selected for each system. The system of the Sun, we have seen, prescribes the constant 1·7275 and the approximate form of the curve. Since there are seven satellites of Jupiter (counting VI. and VII. as one) and nine of Saturn, we may say that we find it possible



This curve represents the $f(\theta)$ of the general expression given in the text. Saturn's rings may be treated in several different ways, but they always come into nearly the same place.

The formula makes the slope steeper between 80° and 90° than it is drawn above.

to adjust sixteen points to this curve by using eight selected constants.

9. Since the scale of the curve can be chosen to be what we like, by suitably altering all the constants A; and since, further, the minimum can be chosen to be anything we like, by altering all the constants B; let us take the minimum as zero and the maximum as unity. Then the factor in brackets varies from B to B + 1: and the ratio

$$\max/\min = (B+1)/B \quad \text{or} \quad B = \min/\max - \min.$$

The quantity B is thus a mere number definitely determinable for each system: but the quantity A which contains the unit of distance, and depends on the value adopted for n, is not of so much interest.

10. From empirical curves for the various systems the following values were obtained:—

TABLE V.

System.	Min.	Max − Min = A.	Ratio = B.	α.	β.
Sun . .	0·8430	0·4162	2·025	−0°·8	56°·6
Jupiter . .	0·8375	0·4523	1·852	41·0	36·0
Saturn . .	0·022	3·074	0·0071	78·0	10·0
Uranus . .	0·24	2·98	0·0805	125·7	12·5

11. The values of α and β are given in the table also. It remains to give the curve to which they apply, which is shown in the diagram. Harmonic analysis of this curve gave the following figures:—

$$0{\cdot}4594 + {\cdot}168\cos 2(\theta - 60^\circ{\cdot}4) + {\cdot}053\cos 4(\theta - 77^\circ{\cdot}2) + \cdots$$
$$+ {\cdot}396\cos(\theta - 27^\circ{\cdot}4) + {\cdot}062\cos 3(\theta - 28^\circ{\cdot}1) + {\cdot}009\cos 5(\theta - 22^\circ)$$
$$+ {\cdot}012\cos 7(\theta - 40^\circ{\cdot}4) + \cdots$$

12. The even and odd multiples have been kept separate to bring out the fact (which Professor Turner has pointed out to me) that the odd multiples approximately form the series

$$0{\cdot}4[\cos\psi + \tfrac{1}{6}\cos 3\psi + \tfrac{1}{36}\cos 5\psi + \cdots]$$

where $\psi = \theta - 27^\circ{\cdot}5$ say. (The zero point for θ was taken quite arbitrarily.) And this series again is very nearly

$$\cos\psi/(3 - \cos 2\psi)$$

13. The line of even multiples is not very different from

$$1/\{6 - 4\cos 2(\psi - 30^\circ)\}$$

And if the curve

$$y = \cos\psi/(3 - \cos 2\psi) + 1/\{6 - 4\cos 2(\psi - 30^\circ)\}$$

be formed, it is seen at once to present the salient features of the

empirical one, though they are not quite identical. Table VI. shows the accordance of the empirical curve with the actual distances:—

TABLE VI.

	n.	Log $A(1{\cdot}7275)^n$.	Log $B+y$.	Log Product.	Log Dist.
Mercury	−2	9·145	0·443	9·588	9·588
Venus	−1	9·382	0·477	9·859	9·859
Earth	0	9·619	0·381	0·000	0·000
Mars	+1	9·857	0·326	0·183	0·183
Jupiter	+3	0·332	0·384	0·716	0·716
Saturn	+4	0·569	0·411	0·980	0·980
Uranus	+5	0·8064	0·4772	1·284	1·284
Neptune	+6	1·044	0·435	1·479	1·479
J. V.	−2	9·1806	0·4527	9·633	9·633
J. I.	0	9·655	0·345	0·000	0·000
J. II.	+1	9·893	0·309	0·202	0·202
J. III.	+2	0·1302	0·2744	0·405	0·405
J. IV.	+3	0·368	0·282	0·650	0·650
J. VI. & VII.	+6	1·080	0·360	1·440	1·439
J. VIII.	+7	1·317	0·427	1·744	1·744

	n.	Log. $A(1{\cdot}7275)^n$.	Log $B+y$.	Log Product.	Log Dist.
S. 1	−2	0·013	9·786	9·799	9·799
S. 2	−1	0·250	9·657	9·907	9·907
S. 3	0	0·488	9·512	0·000	0·000
S. 4	+1	0·725	9·382	0·107	0·108
S. 5	+2	0·962	9·290	0·252	0·252
S. 6	+4	1·437	9·180	0·617	0·618
S. 7	+5	1·675	9·026	0·701	0·701
S. 8	+9	2·624	8·459	1·083	1·082
S. 9	+10	2·862	8·780	1·642	1·641
U. 1	0	0·474	9·526	0·000	0·000
U. 2	+1	0·712	9·432	0·144	0·144
U. 3	+2	0·949	9·410	0·359	0·359
U. 4	+3	1·186	9·299	0·485	0·485

Note.—The "y" in the above table is taken from the empirical curve, not calculated from either of the formulæ. If we take the calculated value of the 7-term formula (given above) for each of these 28 points, the *average* difference between the two y's is not quite $\pm 0{\cdot}007$, and in one case only does the difference exceed 0·016. I hope, however, to improve the formula.

14. We now come to an important feature of the curve, which is possibly of physical significance. It can be seen from the curve itself, or from either of the above analytical expressions for it, that there are two points of great slope. The fractional expression given above shows that the slope is nowhere vertical, since neither of the denominators $(3 - \cos 2\psi)$ or $\{6 - 4\cos 2(\psi - 30^\circ)\}$ can ever vanish. But suppose for a moment that the slope were vertical: this would mean that for the same value of n we had more than one available distance of a planet: *i.e.* there might be two or more planets at nearly the same distance from the primary. Now there are two conspicuous cases of this kind; the minor planets, and Saturn's rings: and a third, not perhaps quite so conspicuous, is the twin satellites VI. and VII. of Jupiter. Is there anything in the curve corresponding to these facts? For the last case there is nothing very remarkable: but it is certainly striking that the places for the minor planets and for Saturn's rings are *almost exactly the two places of maximum slope on the curve.*

15. In view of the difficulty of attaining anything like finality with a purely empirical process, it seems better to give at present figures which are only illustrative, though they are the outcome of

very considerable labour. It must be understood also that the precise details of the curve shown in the diagram are subject to further revision. (They have been revised several times already.) It is possible to assume somewhat different values for the maxima and minima of the individual curves, especially in the case of the systems of Saturn and Uranus, and the exact position of the satellites on the curve is affected by any change in these values. But while details may be thus altered, the essential features of the curve remain always the same. Greater certainty in detail might be obtained through the discovery of some satellite at present unknown. The curve will give the approximate mean distance which it should have, if this "law" is correct.

16. The following seems to be a physical interpretation of the facts sketched in this paper:—

If the distances of the planets and satellites obey with tolerable fidelity a regular law which has the same features for systems so different as those of the Sun and Jupiter, this law probably applies to the original distances, and not to the distances as modified by either tidal action or mutual perturbations. In other words, the present distances have been but little altered by either of these causes since the formation of the planets or satellites.

17. Professor Turner points out to me that this conclusion is in complete accordance with that arrived at by Sir G. H. Darwin in 1881 from totally different considerations; and it seems to entitle us to go further in this direction than he was able to go. Sir G. H. Darwin's words are as follows:—

"According to the nebular hypothesis the planets and the satellites are portions detached from contracting nebulous masses. In the following discussion I shall accept that hypothesis in its main outline, and shall examine what modifications are necessitated by the influence of tidal friction.

"In § 7 it is shown that the reaction of the tides raised in the Sun by the planets must have had a very small influence in changing the dimensions of the planetary orbits round the Sun, compared with the influence of the tides raised in the planets by the Sun.

"From a consideration of numerical data with regard to the solar system and the planetary sub-systems, it appears improbable that the planetary orbits have been much enlarged by tidal friction since the origin of the several planets. But it is possible that part of the eccentricities of the planetary orbits is due to the cause.

"We must therefore examine the several planetary sub-systems for the effects of tidal friction.

"From arguments similar to those advanced with regard to the solar system as a whole, it appears unlikely that the satellites of Mars, Jupiter, and Saturn originated very much nearer the present surfaces of the planets than we now observe them. But the data being insufficient, we cannot feel sure that the alteration in the dimensions of the orbits of these satellites has not been considerable. It remains, however, nearly certain that they cannot have first originated almost in contact with the present surfaces of the planets,

in the same way as, in previous papers, has been shown to be probable with regard to the Moon and the Earth.

"The numerical data in Table II., § 7, exhibit so striking a difference between the terrestrial system and those of the other planets, that, even apart from the considerations adduced in this and previous papers, we should have grounds for believing that the modes of evolution have been considerably different."—G. H. D., *Phil. Trans.*, Part ii., 1881.

Some Types of Prominences associated with Sun-spots.

By Mrs. M. A. Evershed. (Plates 13–16.)

(*Communicated by J. Evershed.*)

The discovery of radial motion in sun-spots, made at this Observatory, reveals the fact that in the penumbra of every spot there are two opposite movements; for at the level of the reversing layer the gases are invariably flowing outwards, away from the umbra, while at the higher levels of hydrogen and calcium they are flowing inwards. It seems possible that some light may be thrown upon these remarkable movements by investigating a still higher region, that of the prominences, where moreover any indraught or outflow could be followed to a far greater distance from the spot centre, because the prominences rise above the faculæ which appear to mask the motion at its outer limits at lower levels.

Mr. Slocum has recently published some photographs of a fine prominence group * in which he detected here and there a flow towards the centre of a large spot then on the limb. This, however, hardly seems to prove that all the great masses on either side of the spot were pouring into it, as he suggests, especially as he also saw some "bright jets coming directly out of the spot." This prominence group, which was also photographed at Kodaikánal on the 8th, 9th, and 10th of October 1910, belongs to a type which I have been studying for some time. The structure always does suggest movement, usually of a violent kind, but when one tries to measure any particular streamer which looks as if it must be rushing in or out, it is usually found that it cannot be identified with certainty on a later plate, or that it has remained stationary. Where evidence of motion has been found, it is almost always outwards. The single instance, therefore, quoted by Mr. Slocum is certainly not enough to prove that sun-spots act as centres of attraction for prominences.

The type of prominence alluded to is distinguished by having a definite centre, often marked by an exceedingly brilliant point or short jet rising from the chromosphere; and from this centre streaks and streamers diverge both north and south. Spectroscopic

* *Astrophysical Journal*, vol. xxxvi. 265.

observations often indicate motion in the line of sight also, so that we may conceive the disturbance as radiating in every direction. The streaks are generally beaded, or carry a bright bead at the extremity. Sometimes they are straight, but often they curve, and then many of them may frequently be seen lying nearly parallel and meeting in great banks or cloudy pillars at some distance from the centre.

Sometimes, on the contrary, they stand nearly vertically above the chromosphere, and, abruptly turning, appear to fall again near the same place: then we have the arched or fountain form. The sides of the arches are often beaded, like the streaks, and there is generally a bright condensation at the top. These arch forms are usually transient or in rapid change.

There is also a rare form, in which only one arch appears, and this is remarkably stable, maintaining the same form for hours, or even days. In all the arch forms it is remarkable that they are more or less of the shape of a Moorish, not a Gothic, arch; that is, the sides converge towards the base, like a horse-shoe, instead of diverging.

The common feature of all these prominences is that they have a centre of radiation, so that we may for convenience call them radiating prominences, the commonest types being the streak, the streamer, and the arch forms, while others are quite peculiar. I measured the centres of over sixty, which were photographed here during the years 1908, 1909, and 1910, and found that with the exception of two, which were doubtful, there was always a spot or spot-group crossing the limb at the same latitude on the same day as that on which the prominence appeared. This was the more striking because, in another type of prominence which I had previously examined (massive forms united to the chromosphere by stems), not one had a spot in its neighbourhood. It may then be taken for granted that these radiating prominences are formed just above sun-spots, but by no means above all spots, nor during the whole lifetime of any single spot, for many a spot crosses the limb entirely unattended by any display of prominences.

What then is the nature of the connection between sun-spots and radiating prominences? And especially, since their forms so strongly suggest movement, what is the evidence that gases are rushing into or out of spots at this high level?

In order to present some of the evidence obtained, and also to show the strange and beautiful forms assumed, Mr. Evershed has made a set of slides enlarged directly from the original plates, which illustrate the characteristic features of eleven of these prominences.

Description of Slides. (Plates 13–16.)

On each plate the centre of radiation is marked, showing the latitude and the hour at which the photograph was taken. This is given in Indian Standard Time, which is $5\frac{1}{2}$ hours in advance of Greenwich.

First Series, Nos. 1 *to* 3. East limb. February 25, 1909.

A small but highly characteristic display of beautifully beaded streaks. The fine details are best seen on the negative (No. 4.* the same as positive No. 3).

The three positives are from plates taken at $8^h\ 9^m$, $8^h\ 28^m$, and $10^h\ 34^m$, Indian Standard Time.

In the first two there is a very brilliant triple centre, occupying about 2°, with its mean position at 6° north; in the last these central jets are mere dots, but directly over them are streaks 60″ high. Unless these streaks have suddenly become visible out of nothing, so to speak, they must have rushed up in the last two hours, and it certainly looks as if the centre had blown up. Change may also be noted in the streak to the left (north), which has a bright patch at its extremity: this changes in shape in the later plates, being twisted up so that it becomes a little higher at the tip, although the part attached to the streamer is no higher than before.

This prominence was observed by Mr. Sitarama Aiyar, the first assistant at this Observatory, who saw the C line slightly displaced to the violet in the top streaks.

A stream of small and very active spots, with Greenwich number 6632, was on the meridian March 3·6, and therefore must have crossed the limb on this day, February 25: the mean position of the group was 4° north latitude.

Second Series, Nos. 5 *to* 7. East limb. September 26, 1910.

Another example of radiating streaks, but these are more nearly horizontal to the Sun's surface. The brilliant central jet at 13° south, which here is double, also obviously rises between $8^h\ 20^m$ and $8^h\ 33^m$: the rise amounts to 12,000 km., or 16 km. per sec. The first plate ($8^h\ 20^m$) shows a long streamer which is straight, but later becomes curved: very careful measurement shows that the streamer (which is tipped by a bomb) is a little longer at $8^h\ 33^m$ and is also falling slightly towards the chromosphere. It appears to be travelling out from the centre at a rate of 2·7 km. per sec., and falling at about the same speed.

The centre from which these streaks certainly appear to be shooting forth coincides with the centre of spot 6893 (Greenwich number), for its latitude was 13° south and it crossed the central meridian October 2·2. When the group went off the west limb, it was accompanied by the display photographed by Mr. Slocum.

Third Series, Nos. 8 *and* 9 (The Lady); *Nos.* 10 *to* 12 (The Lady gone). East limb. December 22 and 23, 1909.

The interesting feature on December 22 is a curved streamer with a bright condensation at its highest point. The fainter end curves very nearly down to the chromosphere, where a bright

* Not reproduced.

point nearly meets it; in the second plate ($8^h\ 32^m$) this down-curving portion has gone. Has it been sucked up and drawn into the brilliant centre, or has it fallen off? The gradually curving and dropping streamer of the last series inclines us to take the latter view. The other streamers are baffling: one grows longer, another shorter, in the 12 minutes' interval between the two plates. The streamers are exquisitely beaded, especially on the first plate, which is the best. At a little distance is a tall prominence like the figure of a lady looking on.

On the following day there are more eruptions, but the lady has vanished. At $8^h\ 45^m$ there is an extremely brilliant bomb, and if it is identical with the highest bomb seen at $10^h\ 42^m$, it has risen 25,000 km., or at the rate of 3·7 km. per sec. In the first plate of this day ($8^h\ 13^m$) note an arch over latitude $-15°$. This marks the place of a spot which on its return was heralded by a magnificent display of arches, which we shall show presently.

Mr. Nagaraja Aiyar observed a displacement to the red in this prominence for 1·5 Å on the 22nd, and 5·0 Å on the 23rd, implying movements in the line of sight of about 70 to 200 km. per sec.

A large regular spot, with some small attendants (Greenwich No. 6791), crossed the east limb at this time with mean position 10° north. On December 24 it was observed here that C was reversed and D_3 dark in its vicinity. It crossed the central meridian on December 29·1, G.M.T.

Fourth Series, Nos. 13 *to* 15. East limb. April 28, 1910.
(The Cauliflower.)

It is only necessary to glance at this series of three to see that the bundle of bombs was rising, and Mr. Sitarama Aiyar's notes show that there was also movement in the line of sight, for he saw displacements over almost the whole prominence in both directions, C and D_3 appearing double in several places. As the interval between the last two plates is much greater than between the first two, the motion is more striking to the eye, but it was really diminishing in velocity. Measurements of the photographs show that the small bright spot just above the cauliflower-like mass moved upward at first at a rate of 12 km. per sec., and later at 3·6 km. per sec. The main mass was not moving quite so fast as the spot at first (8·2 km. per sec.), but caught it up and is seen touching it, because the spot grew larger; afterwards both were moving at nearly the same velocity (the spot 3·6, the main mass 3·2 km. per sec.).

A new group of sun-spots was visible close to the east limb on the day following this extraordinary outburst, and the main spot was on the parallel of latitude 2° north. It was very active on the 30th of April. When going off west, it was accompanied by a number of large irregular arches, 90″ high. The Greenwich number was 6842: it did not reappear.

Fifth Series, Nos. 16 *and* 17. West limb. January 9 and 10, 1909. (The Telegraph Post.)

We now come to examples of curving streamers radiating from a centre and merging at their other extremity in banks or columns. This group is specially well developed on the south side, for there is often a lack of symmetry of this kind. The centre (at 9° north) is very brilliant, and was higher on a plate taken an hour later (not shown), so that here again we have a plain instance of gas welling up from the spot-centre. On the following day the single broad jet has vanished, but there are two bright points at 9° and 11½°. The rather small mass seen to the south at $8^h\ 20^m$ on the 9th was higher and thicker at $9^h\ 19^m$, and streamers extended from it in both directions. On January 10, $8^h\ 32^m$, it will be observed that it has greatly developed; it is now leaning away from the centre, and the top is drawn out into a horn pointing away south; a great number of fine streamers run between it and the centre, like wires from a telegraph post. Is this tall post pouring its matter into the spot along the "wires," or is matter shooting out from the spot, and building up the post? There is nothing on the plates which helps us to determine the question, except that gas has undoubtedly been rising from the centre, and the post has not diminished but greatly grown both in height and thickness. Is not this merely a later stage of something like what we saw in our series 2 (September 26, 1910), where one streamer at least was shooting outwards and carrying its bomb gradually downwards towards the chromosphere?

The spot with which this prominence was connected was a very large one, followed by numerous small companions: its mean position was 9° north. It crossed the central meridian January 3·3 (G.M.T.), and had Greenwich No. 6591.

Sixth Series, Nos. 18 *and* 19. West limb. October 9, 1910. (Mr. Slocum's Spot.)

A splendid confusion of beaded streaks and streamers, short and long, straight and curved, with masses and filaments, mingled together on both sides, the whole extending for quite 45°, with its centre at 12° south. We have examined the plates very carefully for motion, but can find none. One long, nearly horizontal streamer (seen on the left) is 8000 km. shorter towards the spot in the second plate, rather suggesting motion outwards than inwards. Mr. Slocum found motion in both directions on his photographs, which were taken about twelve hours earlier than ours of the 9th, and twelve hours later than ours of the 8th.

The spot was a double group forming a splendid stream visible to the naked eye. It is the same group which was accompanied by the curving and dropping streamer of our second series when it crossed the east limb on September 26, 1910. This was its third apparition, when it received the Greenwich numbers 6893 and 6894. The mean latitudes of the double group were 13° and 15° south.

Seventh Series, No. 20. West limb. September 22, 1909. (The Wing.)

This is rather a rare type, showing only a slight development to the north, from a centre at 10° north, but a massive form south, spreading over the equator like a great bird's wing from 10° north to 10° south. It is joined to the centre by two slender filaments. The exceedingly definite edge of the mass southwards is a peculiar feature, but by no means unknown in other types. An earlier plate at 8^h 33^m, and several later ones, show so little difference that we have not reproduced them: the mass shows no upward or downward movement, but there is slight outward movement between 8^h 33^m and 8^h 55^m, amounting to about 2 km. per sec. This was found by measuring the distance in each plate between the central point and the distinct edge of the southern mass.

This prominence seems to have been connected with quite a new spot, which was seen at Kodaikánal to be forming in this region on September 20 (two days before the prominence appeared): it returned to the visible disc as a large regular spot at latitude 10° north, followed by small companions, and crossed the central meridian October 13·1 as Greenwich No. 6741. Its earlier appearance near the west limb does not seem to have been noted at Greenwich.

Eighth Series, Nos. 22 *to* 26. West limb. December 6 and 7, 1909. (The Cockroach.)

Another peculiar form. The negative (No. 21),* enlarged from a photograph taken at 9^h 16^m December 6, shows all the delicate detail of this curious prominence, which has the appearance of a gigantic insect with many legs. In the series of positives (three for December 6) the insect seems to be trying to fly away, but is so firmly held by threads to the bright low mass at 15° south that it does not move, either upwards or outwards, towards the south. The chief changes on this day consist in the legs becoming thicker and more numerous.

Next day it is much smaller, and seems to have succeeded at last in soaring upwards, having almost completely broken its connection with the bright prominence at 15° south, though it is still guyed down to the chromosphere vertically below it by filaments. New threads are put out towards the bright prominence at 10^h 19^m.

It is curious that this soaring and diminishing prominence should be the last farewell of a large spot, which had appeared no less than four times. Its Greenwich number on this occasion was 6772, and it had crossed the central meridian on November 28, its mean position being 13° south. When it went off the west limb for the first time, on September 14, it was accompanied by a few prominences; and when it came on to the east limb for the

* Not reproduced.

second time there was a typical display of streamers and bombs (September 27 and 28). There were no displays in October and November, but it was monsoon time and the plates were often poor.

Ninth Series, Nos. 27 to 30. West limb. November 19, 1908. (The Arch.)

We now come to the arched forms, which are fairly frequent, though not so common as the streamers, streaks, and bombs. This particular prominence remained at a constant height, though it changed in shape, and at $9^h\ 29^m$ almost vanished (though the definition was still good), to return at $10^h\ 19^m$. Another faint arch was meanwhile taking form beside it. The curious disappearance may have been due to a displacement of the K line from the second slit, owing to motion in the line of sight.

Here, as often, there is change of intensity but no evidence of motion. In another case, however, an arch distinctly rises, and at the same time a low jet rushes up into one of its sides. (This was photographed on January 22, 1909: the forms are too delicate to show well on the screen.)

On the previous day there were streamers and a bank with a bright centre at 12° north at this place.

This is the first of the series of prominences which accompanied spot-group 6564 (position 13° north) in some of its seven apparitions; it was first seen, as a pair of small spots, on November 7, but quickly developed into a splendid group visible to the naked eye.

Tenth Series, Nos. 31 to 36. West limb. January 7, 1910. (Fountains.)

An ascending movement is clearly shown in a magnificent series of arches photographed fifteen times on January 7, 1910. This display accompanied the spot which was marked at its crossing of the east limb by the feeble arch shown on a plate of our third series (December 23, 1909, $8^h\ 13^m$, No. 10). In the first plate, at $8^h\ 5^m$, there is only one group of arches, and the form is scarcely distinguishable: they might be mere streamers with bright heads, but later they appear distinctly as arches, seen edgewise, and the bright tops rise with diminishing velocity.

Between $8^h\ 5^m$ and $8^h\ 53^m$ the rate of ascent is 3 km. per sec.

Between $8^h\ 53^m$ and $10^h\ 10^m$ the rate of ascent is 1·9 km. per sec.

At $10^h\ 31^m$ and $11^h\ 6^m$ the height remains the same.

Between $11^h\ 6^m$ and $12^h\ 44^m$ the rate of rise is 1·5 per sec. At this time they seem to have floated free from their base, but part of the K line may have been thrown off the slit, through change of temperature in the spectroheliograph, combined with distortion of the K line due to motion.

This is the only group of arches at $8^h\ 5^m$, but at $8^h\ 53^m$ we see some streaks beside it, and at $10^h\ 10^m$ there is a second group,

fully formed, interlacing and beautifully beaded, wide open, so that we look through them. As they rose to their full height in the space of two hours and five minutes the average rate of ascent would be 12 km. per sec.

Mr. Subrahmanya Aiyar noted that at $9^h\ 10^m$ the C line was displaced to the red at the top and to the violet at the base of the arches, while eight minutes later it was displaced to the red at the base for 2 Å, and to the violet over the rest of the prominence.

The spot with which these arches and the single arch of December 23 were associated was No. 6793, with position 15° south. It was a large active spot, with a few small companions.

Eleventh Series, Nos. 37 *to* 42. East limb. July 31, 1908. (Ring.)

An extremely brilliant and stable prominence, which had the same form on the previous day, when it was photographed on an excessively poor flocculus plate through thick cirrus. No plate could be taken on August 1 or 2, because of cloudy weather.

It is here shown on two fast-exposure disc plates, and its shape is quite clear, even though the first photograph was taken (at 8 a.m.) through thin cloud. These plates show how it rose directly out of a large flocculus, and they show plainly the usual bright condensation at the top of the arch, which is not so clear on the longer-exposed prominence plates. This arch has a horse-shoe shape, and its appearance almost suggests that it was continued on the chromosphere as a complete ring. The four selected prominence plates show that though the ring did not alter in height during the time it was observed here, it changed in width, being rounded and thick in two plates, and in the two alternate plates elliptical and thinner. We have tried to deduce motion from the two small jets which stand out on each side, like two little ears; the one on the left (north) side certainly seems to be descending between $8^h\ 47^m$ and $9^h\ 25^m$.

It is gratifying to find confirmation of this in the very interesting observations of Father Chevalier at Zô-zè.* He was fortunate enough to see this remarkable prominence well, and his observations of July 31, which began two hours earlier than ours, showed it to him at first as a much lower arch with the northern side incomplete, not reaching the chromosphere; in an hour's time it was higher and a perfect horse-shoe arch. Riccò adds a note that he saw it as a curved jet on the 29th, and, persisting as an arch on the 30th and 31st, disappearing on the 1st of August.† Both observers were struck with its extraordinary brilliancy, and note that the brightest part was the summit. Father Chevalier found that this part gave a continuous spectrum on the 30th.

Putting all these observations together, it seems clear that this arch originated in an uprush of intensely hot gas which curved

* *Memorie della Società degli Spettroscopisti Italiani*, vol. xxxviii. p. 17.

† "Il 10 agosto" is, I presume, a misprint for "il 1".

and fell northwards, and that a stream continued to ascend and follow nearly the same course for three days. The bright condensation at the summit, so characteristic of these arched forms, remains unexplained, for Father Chevalier's suggestion does not commend itself to us as satisfactory.

The spot over which this ring stood was oval, crossed by bright bridges, and was large enough to be seen by the naked eye. This was its second apparition. When it went off the west limb on August 14, it was entirely unaccompanied, though there were some insignificant prominences in the place the day before.

In conclusion, our investigation suggests more problems than it solves, yet some preliminary conclusions may be drawn.

1. The movements observed in these prominences, which are situated directly above sun-spot groups, are of quite a different kind from those discovered in the penumbræ of spots; for, instead of being uniform and constant, they are intermittent, and vary both in direction and amount.

2. In the greater number of radiating prominences we have been unable to find direct evidence of motion, but where found the motion is outward from the centre.

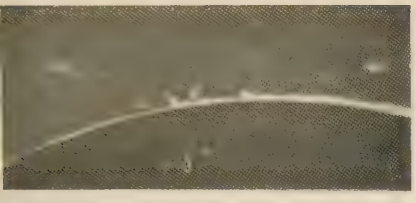
3. The visible movements are slower than one would expect both from the appearance of the prominences and from the movements observed in the line of sight, which are occasionally of the order of a hundred km. per sec. The gas has several times been observed to rise from the centre at a rate of about 12 km. per sec., but this velocity diminishes; at a distance from the centre it is of the order of 2 or 3 km. per sec.

4. The outward-moving gas seems to have a tendency to fall back upon the chromosphere. Sometimes it apparently accumulates there, and forms massive banks, but sometimes it seems to rise and fall again continuously like a fountain. Occasionally it rises very high and disintegrates.

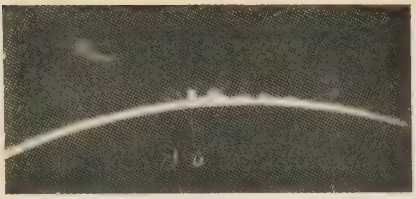
There can be no doubt that other forces are at work on the Sun's surface besides an eruptive force and gravity. Prominences of very varied forms are seen falling, rising, expanding horizontally, being checked and driven back, stretching out long filaments towards the chromosphere or towards each other.* Light pressure, electric forces acting on ionised gases, and the magnetic force which we know from the discoveries of Professor Hale must exist in the vicinity of sun-spots, all must produce their own effects, and they must mask the upward and outward movements, which nevertheless do seem to be traceable to the intermittent action of certain sun-spots upon the gases immediately above them.

* The rising prominences in some cases observed at Kodaikánal move with an *accelerating* velocity and appear to be driven entirely away into space by a force opposed to gravity (see *Astrophysical Journal*, vol. xxviii. p. 79).

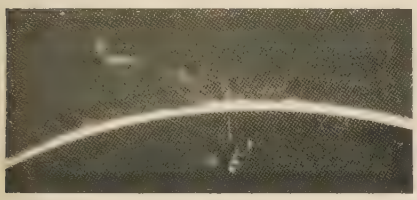
The Observatory,
Kodaikánal, South India:
1913 March.



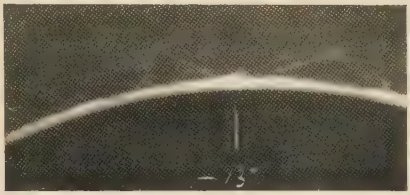
1. E.L. 1909 Feb. 25, $8^h\ 9^m$.



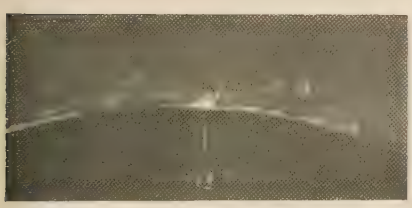
2. E.L. 1909 Feb. 25, $8^h\ 28^m$.



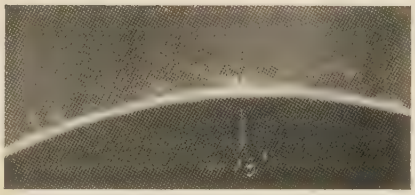
3. E.L. 1909 Feb. 25, $10^h\ 34^m$.



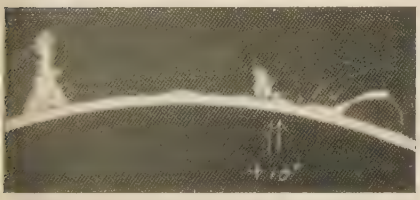
5. E.L. 1910 Sept. 26, $8^h\ 20^m$.



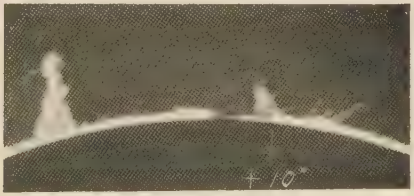
6. E.L. 1910 Sept. 26, $8^h\ 33^m$.



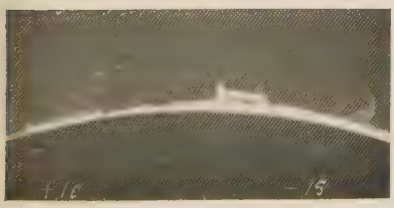
7. E.L. 1910 Sept. 26, $9^h\ 13^m$.



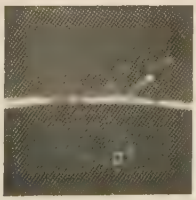
8. E.L. 1909 Dec. 22, $8^h\ 20^m$.



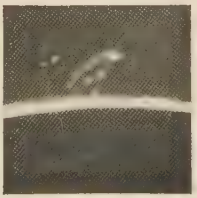
9. E.L. 1909 Dec. 22, $8^h\ 32^m$.



10. E.L. 1909 Dec. 23, $8^h\ 13^m$.

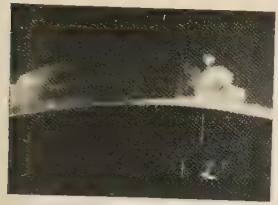


11. E.L. 1909 Dec. 23, $8^h\ 45^m$.



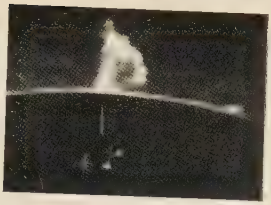
12. E.L. 1909 Dec. 23, $10^h\ 42^m$.

TYPES OF PROMINENCES. MRS. EVERSHED.

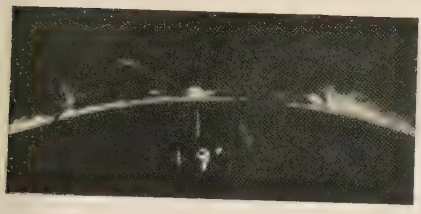


13. E.L. 1910 April 28, $8^h\ 58^m$.

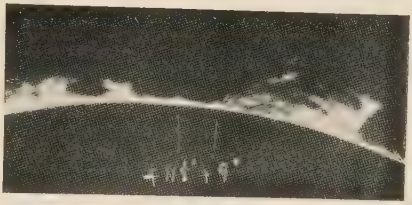
14. E.L. 1910 April 28. $9^h\ 19^m$.



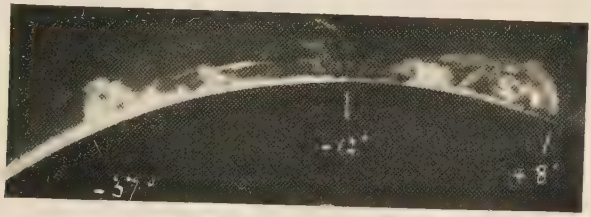
15. E.L. 1910 April 28, $10^h\ 27^m$.



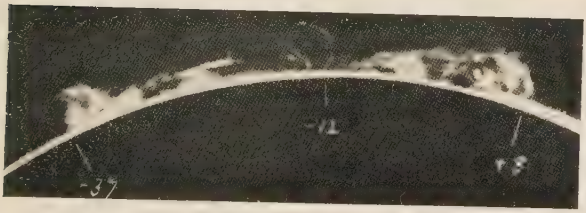
16. W.L. 1909 Jan. 9, $8^h\ 20^m$.



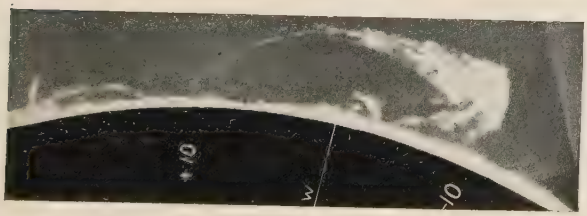
17. W.L. 1909 Jan. 10, $8^h\ 32^m$.



18. W.L. 1910 Oct. 9, $9^h\ 56^m$.

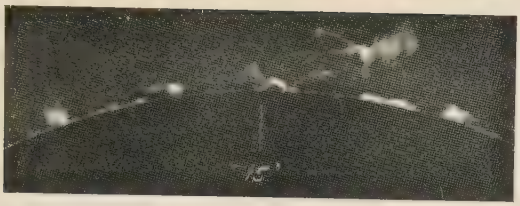


19. W.L. 1910 Oct. 9, $10^h\ 11^m$.



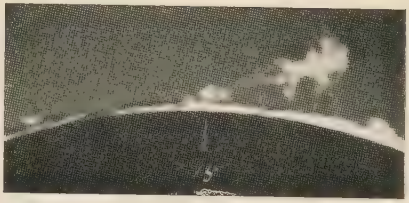
20. W.L. 1909 Sept. 22, $8^h\ 55^m$.

TYPES OF PROMINENCES. MRS. EVERSHED.

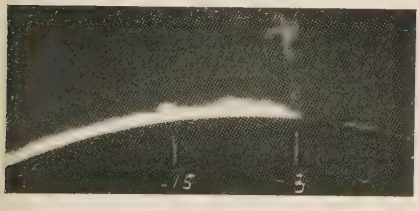


22. W.L. 1909 Dec. 6, $8^h\ 23^m$.

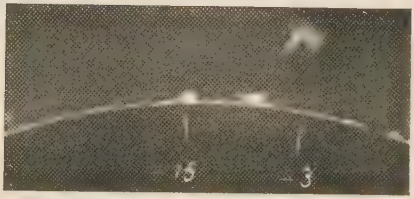
23. W.L. 1909 Dec. 6, $9^h\ 16^m$.



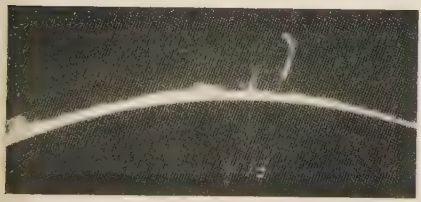
24. W.L. 1909 Dec. 6, $10^h\ 19^m$.



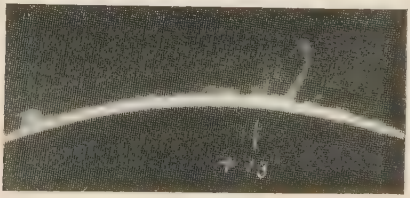
25. W.L. 1909 Dec. 7, $8^h\ 51^m$.



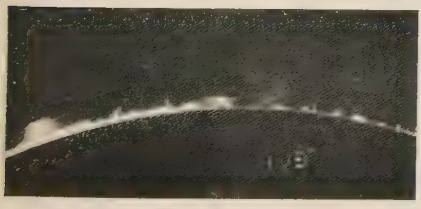
26. W.L. 1909 Dec. 7, $10^h\ 19^m$.



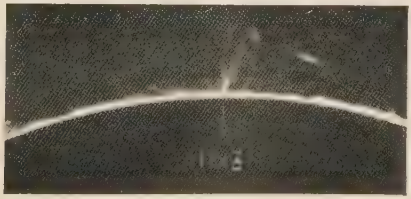
27. W.L. 1908 Nov. 19, $8^h\ 25^m$.



28. W.L. 1908 Nov. 19, $8^h\ 33^m$.

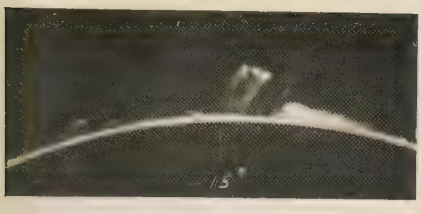


29. W.L. 1908 Nov. 19, $9^h\ 29^m$.

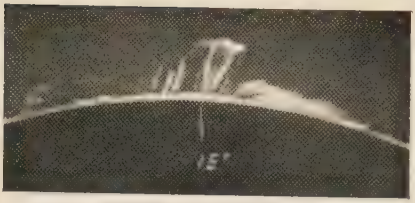


30. W.L. 1908 Nov. 19, $10^h\ 19^m$.

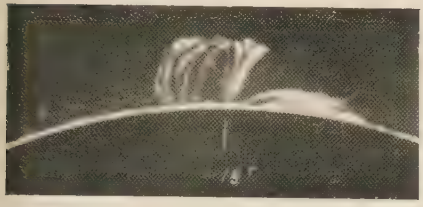
TYPES OF PROMINENCES. MRS. EVERSHED.



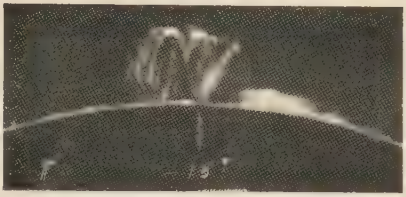
31. W.L. 1910 Jan. 7, 8^h 5^m.



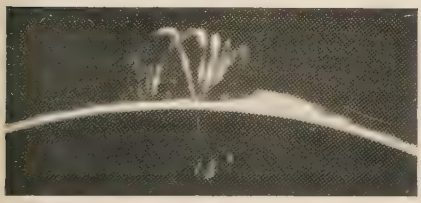
32. W.L. 1910 Jan. 7, 8^h 53^m.



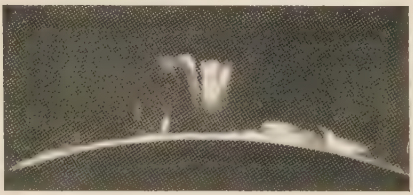
33. W.L. 1910 Jan. 7, 10^h 10^m.



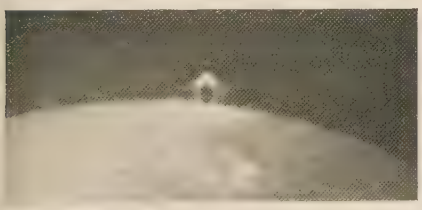
34. W.L. 1910 Jan. 7, 10^h 31^m.



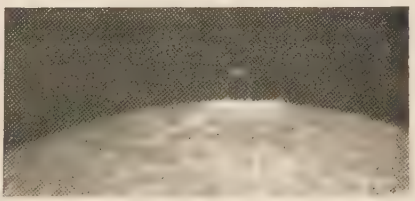
35. W.L. 1910 Jan. 7, 11^h 6^m.



36. W.L. 1910 Jan. 7, 12^h 44^m.

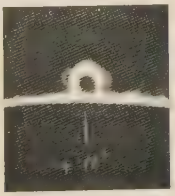


37. E.L. 1908 July 31, 8^h 0^m.



38. E.L. 1908 July 31, 9^h 18^m.

39. E.L. 1908 July 31, 8^h 47^m.



40. E.L. 1908 July 31, 9^h 7^m.

41. E.L. 1908 July 31, 9^h 25^m.

42. E.L. 1908 July 31, 9^h 34^m.

TYPES OF PROMINENCES. MRS. EVERSHED.

Sun-spots and Terrestrial Magnetic Phenomena, 1898–1911 : *Sun-spot Areas, Magnetic Storms, and the Sun's Corona.* (Third Paper.) By the Rev. A. L. Cortie, S.J.

1. A connection in general between the stream lines of the solar corona and magnetic storms has been suggested by other investigators. In his preliminary report of the Indian eclipse of 1898 (*Monthly Notices R.A.S.*, lviii., Appendix, pp. [40] [41]), Sir Norman Lockyer wrote: "The violent magnetic storm and bright aurora on March 15 and 16, to which Dr. Chree has recently called attention, follow suit with the chemical and eclipse observations, and it is important to note, as Dr. Chree has informed me in a later communication, that there were less violent disturbances on January 15–18 and February 11–16, so that there have been three disturbances separated roughly by an interval of twenty-eight days."

Mr. Maunder treats of the subject in rather more detail in his paper on "Magnetic Disturbances and their Association with Sun-spots" (*Monthly Notices R.A.S.*, lxv. pp. 32, 33), and calls especial attention to the four long rays photographed by Mrs. Maunder in the same eclipse, and to the significance of their rising from synclinal structures on the Sun's limb, each structure terminating in a long, rod-like ray. But so far no detailed analysis has been attempted to trace a connection, if any such exists, between disturbed areas on the solar surface, which have been associated with magnetic storms, and the rays of the solar corona. The lines of the present inquiry, which are limited to the years 1898–1911, are as follows: first, to find whether, at the time of total solar eclipse, a region of sun-spot activity was on or near the limbs of the Sun; secondly, whether such a region of spot-activity, if found, was associated with notable magnetic activity on the Earth during its life-history; and thirdly, whether there was evidence of marked disturbance in the solar corona, connected with any such region of associated sun-spot and magnetic activity.

The coronæ of the eclipses of 1900, 1901, and 1911 of the minimum type have naturally no place in the investigation, as the number of magnetic disturbances in each of these three years was small, greater magnetic storms being entirely absent, and the mean daily range of the declination magnet, 9′·8, 9′·1, and 12′·7 in each of the three years respectively, indicating a low ebb of terrestrial magnetic activity.

2. *The Total Solar Eclipse of* 1898, *January* 22.—The heliographic longitude of the central meridian of the Sun on the day of the eclipse was approximately 33°, and hence that of the west limb was 123°. A disturbed area on the Sun's surface occupied this longitude, with a mean heliographic latitude −12°·5, from January 19 to May 1, 1898, with a slight recrudescence of activity in the middle of July. The disturbed region was confined within

a belt covering about 15° in longitude and 5° in latitude. A region very close to this in position had been intermittently disturbed in 1897 from January 1 to August 3. The following list contains the Greenwich numbers and the mean positions of the groups of spots concerned, with the dates of their appearance and disappearance, extracted from the Greenwich volumes of sun-spot data.

Group.			Longitude.	Latitude.	
	1897.				
4494	Jan. 1	...	111·9	−12·5	Three preliminary disturbances.
4592	July 3	July 14	115·9	−11·0	
4604	July 31	Aug. 3	120·9	− 8·8	
	1898.				
4680	Jan. 19	...	127·6	−13·6	
4693	Feb. 16	Feb. 17	123·8	−11·6	
4702	March 6	March 17	119·5	−13·2	A series of recurrent groups.
4713	March 31	April 13	128·4	−11·4	
4714	April 6	April 9	117·8	−13·7	
4723	April 29	May 1	130·3	−11·3	
4758	July 19	July 22	132·7	−11·6	
4763	July 29	...	120·9	− 8·8	

After a space of about 10° in longitude there followed, some 4° higher in latitude, a second area of disturbance, restricted to a belt of 15° in longitude, and about 2° only in latitude, the group 4676, mean longitude 101·3° and mean latitude −14·8, being a connecting link between these two closely related areas of disturbance. This second area was marked by the following groups:—

			Longitude.	Latitude.	
4679	Jan. 18	Jan. 24	90·4	−7·8	
4687	Feb. 7	Feb. 19	100·6	−7·1	
4688	Feb. 9	Feb. 20	89·5	−8·4	Recurrent with 4679.
4704	March 7	March 18	104·1	−7·9	Recurrent with 4687.

The following table contains a list of the magnetic storms, moderate (*m*), great (*g*), and very great (*vg*), associated with the groups of sun-spots occurring in these two disturbed areas in the year 1898. The very great magnetic storm of 1898 March 15 associated with the spot-group 4702 is a member of the series. There is some uncertainty whether the moderate magnetic disturbances of July 19–27 are all connected with groups 4758 and 4763. Nevertheless, there is a series of magnetic disturbances connected with these disturbed sun-spot areas during eight synodic rotations of the Sun.

TABLE I.

Associated Magnetic Disturbances, 1898 *January* 11–*July* 31.

Spot-Groups . . .	4676 Jan. 13–21.							4679 Jan. 18–24.							I.
Date . . . Jan.	11	12	13	14	15	16	17	18	19	20	21	22	23		
Magnetic disturbance .					*m*	*m*	*m*	***m***	*m*	*m*	*m*				
Spot-Groups . . .	4687 Feb. 7–19.							4688 Feb. 9–20.							II.
Date . . February	7	8	9	10	11	12	13	14	15	16	17	18	19		
Magnetic disturbance .				*m*	*g*	*g*	***m***	*g*	*m*	*m*	*m*				
Spot-Groups . . .	4702 March 6–17.							4704 March 7–18.							III.
Date . . March	6	7	8	9	10	11	12	13	14	15	16	17	18		
Magnetic disturbance .						*m*			*g*	*vg*	*m*	*m*	*m*		
Spot-Groups . . .	4713 March 31–April 13.							4714 April 6–9.							IV.
Date . . April	2	3	4	5	6	7	8	9	10	11	12	13	14		
Magnetic disturbance .			*m*		*m*	*m*	***m***	*m*	*m*		*g*	*m*	*m*		
Spot-Groups . . .	4723 April 29–May 1.														V.
Date . . April–May	29	30	1	2	3	4	5	6	7	8	9	10	11		
Magnetic disturbance .			*m*		*m*	*m*	*m*						*m*		
Spot-Groups . . .	Region dormant.														VI.
Date . . May–June	26	27	28	29	30	31	1	2	3	4	5	6	7		
Magnetic disturbance .			*m*	*m*	*m*	*m*	*m*					*m*	*m*		
Spot-Groups . . .	Region dormant.														VII.
Date . June–July	22	23	24	25	26	27	28	29	30	1	2	3	4		
Magnetic disturbance .				*m*	*m*	*m*		*m*							
Spot-Groups . . .	4758 July 19–22.							4763 July 19.							VIII.
Date . . . July	19	20	21	22	23	24	25	26	27	28	29	30	31		
Magnetic disturbance .	*m*	*m*	*m*	*m*	*m*	*m*	*m*		*m*						

With regard to the solar corona of 1898 January 22, the base of the streamer which terminates in the long, rod-like ray in the S.W. quadrant covers the spot-area associated with the magnetic disturbances. Assuming the long ray to spring from the limb, its heliographic longitude would be 119° and its latitude −34·8° S. according to measures kindly communicated to me by Mr. Maunder. Its latitude is therefore much lower than that of the spot-area. But above it is a set of equatorial rays which, if they converge near the limb, which they seem to do, would not be very far removed in latitude from the centres of spot-activity. These rays are well shown in the photograph taken by Major Hills, and in the composite drawing made by Mr. Henry Cousens from the photographs taken by the party of observers under the direction of Mr. Naegamvala at Jeur. These convergent equatorial rays only occur on the W. limb, that affected by the centres of spot-activity, and are absent from the E. limb. There seems, therefore, to be the evidence of approximate position to connect these converging rays with the regions of the sun-spots which were associated with the

magnetic storms. One of the coronal masses photographed in the green ray λ 5303 by Professor W. W. Campbell (*Astrophysical Journal*, xi. p. 231) is also coincident in latitude and longitude with the centres of spot-activity.

3. *The Total Solar Eclipse of* 1905 *August* 30.—In my report of the observations of this eclipse to the Royal Irish Academy (*Transactions*, xxxiii., A, pp. 21, 22), attention was directed to the convergence of a set of short streamers or rays to a point on or near the east limb of the Sun situated in latitude − 21° S. The longitude of this limb on the day of the eclipse was 23°. In a subsequent paper, "On the Connection between Disturbed Areas of the Solar Surface and the Solar Corona" (*Astrophysical Journal*, xxiv. p. 358), the origin of the rays was attributed to a spot-group which on the day of the eclipse was removed about 41° from the Sun's east limb, and had a mean latitude − 20° S. The life-history of the group is set forth in the following table of recurrent appearances of the group from May 11 to September 9, derived from the Greenwich results:—

Group.	1905.		Longitude.	Latitude.	
5529	May 11	May 22	63°·8	−15°·2	Recurrent groups.
5553	June 8	June 19	52·3	−14·9	
5586	July 5	July 17	52·4	−15·2	
5608	July 31	Aug. 11	66·7	−12·4	
5611	Aug. 1	Aug. 12	60·8	−16·3	
5639	Aug. 28	Sept. 9	61·0	−18·5	
5550	June 7	June 18	69·9	−15·7	Two associated groups.
5585	July 5	July 7	72·0	−16·7	

These groups, therefore, constitute a disturbed belt on the solar surface extending some 20° in longitude and 7° in latitude. Both the solar-surface disturbance and the coronal disturbance are similar to those studied in the case of the total eclipse of 1898. Although the year 1905 was a year of maximum area of sun-spots, there were relatively but few magnetic disturbances, and not one of the greater type. Nevertheless, 17 disturbances of a total of 45, or about 36 per cent., can be arranged in synodical series associated with this disturbed area on the Sun. It will be unnecessary to repeat a table similar in all respects to Table I. With regard to the convergent coronal rays, they only occur on the east limb, near which the greater number of spot centres were grouped on the day of the eclipse. An alternative focus of convergence might be postulated for the region longitude 350°, latitude − 16°, the position of the great February spot. This would connect the rays with the area which was disturbed intermittently from 1903 April 19 to 1907 April 13, and was associated with a great number of magnetic storms, including two of the very first order, 1903 October 31 and 1907 February 9. But the identification with the region tabled above seems to be the more likely.

4. *The Total Solar Eclipse of* 1908 *January* 3.—Here again there is an example of a set of rays in the corona apparently converging to a focus at or near the Sun's east limb. In Mr. F. K. McClean's account of his expedition to Flint Island, there is a beautiful composite drawing from the photographs of the corona made by Mr. Wesley, which is reproduced in plate 11 of the memoir, and in his description of the corona Mr. Wesley writes (*loc. cit.*, p. 11): "The equatorial rays on the west side present no features calling for special remark. On the eastern side there is more evidence of disturbance; the detail is more defined, and of a more interesting character. There is a mass of rays, near the equator, reaching to a height of 10′ and more, which appears to radiate from a point near the limb." The longitude of the east limb on the day of the eclipse was 245°, and a region of spot-activity near this longitude and just north of the equator existed from 1907 May 6 to 1908 January 8, as is shown in the following table:—

Group.			Longitude.	Latitude.	
	1907.		°	°	
6185	May 5	May 15	245·5	+4·9	
6223	July 24	Aug. 4	251·8	+7·5	
6259	Sept. 17	Sept. 28	243·3	+7·2	
6282	Oct. 13	Oct. 25	253·8	+5·7	
6301	Nov. 9	Nov. 20	258·7	+7·1	Return of 6382.
6305	Nov. 12	Nov. 14	240·3	+8·4	
6306	Nov. 17	Nov. 22	254·8	+9·1	
6321	Dec. 8	Dec. 17	253·3	+7·9	Return of 6301.
	1908				
6340*	Jan. 5	Jan. 8	255·1	+8·4	Return of 6321.

We have here another instance of a belt of disturbance covering nearly 20° in longitude and about 5° in latitude, similar to those associated with converging masses of streamers in the coronæ of 1898 and 1905. It will be again unnecessary to tabulate the associated magnetic storms. They form groups recurring at each synodical rotation of the disturbed spot-area, even when the spots had disappeared and the area was, so far as spots were concerned, seemingly dormant. Fourteen greater storms are included in the list. In my second paper of this series (*M.N.*, *R.A.S.*, lxxiii. p. 153), from a study of the greater magnetic storms, and from the facts that a spot can affect the Earth magnetically when it is removed at a considerable distance from the central meridian, and likewise that magnetic storms associated with a spot frequently occur in batches, it was suggested that not a single stream of particles emanated from a centre of spot-activity, but several divergent streams centred on the same focus of disturbance. The present study of the associated centres of spot-activity, and magnetic storms, and the solar coronæ of 1898, 1905, 1908, would justify that conclusion, for in each of these three cases there existed a belt of

long-enduring spot-activity, associated with synodical series of magnetic disturbances, and in the coronæ masses of rays apparently converging to the centres of disturbance. Two different lines of argument have therefore led to consonant results.

The following summary embodies the conclusions contained in the three papers concerned with this research on the relations between terrestrial-magnetic and sun-spot activity for the period 1898–1911 (*M.N.*, *R.A.S.*, lxxiii. pp. 52–60, pp. 148–155):—

1. In the period there was a lag of the maximum of frequency of magnetic storms upon sun-spot area, and an inversion of the phenomena in the latter part of the cycle, when sun-spots were congregated about the zones $\pm 10^\circ$ of heliographic latitude.

2. After the maximum of a sun-spot cycle the Earth is more favourably situated relatively to the zones of spot-activity, depending on the inclination of the Sun's axis to the ecliptic.

3. The minima in the curve of annual variation of magnetic storm frequency occur near the nodes of the Sun's equator, and the maxima near points removed 90° from the nodes.

4. The inclination of the Sun's axis to the ecliptic, and the position of the Earth relative to the sun-spot zones, would cause such an inequality.

5. In the period 1889–1911 there is a much greater amplitude of variation in the portion of the curve corresponding to the months from the descending to the ascending node, than for those from the ascending to the descending node, and in these years the southern hemisphere of the Sun was the more affected by sun-spots.

6. A greater magnetic storm is independent of the area of the associated sun-spot and of its position relatively to the central meridian.

7. Spots associated with greater magnetic storms are marked by continual change and by irregularity of structure.

8. A general state of magnetic activity corresponds with a general state of sun-spot activity, but the advent of a large disturbed spot, or of one favourably situated in heliographic latitude relatively to the Earth, precipitates a great magnetic storm.

9. Not one, but several magnetic storms, mark the passage of an active spot across the Sun, suggesting that the mode of action of a spot is by a series of streams diverging from a disturbed area.

10. In the eclipses of 1898, 1905, 1908 there is good evidence that long-enduring belts of sun-spot activity, which were associated with synodical series of magnetic storms, are marked in the solar corona by converging sets of streamers.

Stonyhurst College Observatory:
1913 *April* 8.

The Parallax of Nova Geminorum (2), 1912.

By Frederick Slocum.

Nova Geminorum (2) was discovered by Enebo March 12, 1912. It was then nearly 90° east of the sun, and a series of observations with the 40-inch refractor of the Yerkes Observatory for the investigation of its parallax was immediately begun.

Fifteen plates were secured at three parallactic epochs as shown by Table I.

TABLE I

Plates of Nova Geminorum (2). *Taken with the 40-Inch Refractor of the Yerkes Observatory.*

Number.	Date.	Hour Angle. h	Observers.	Quality of Images.
702	1912 Mar. 18	−0·2	Sl., Sl.	Fair
703	Mar. 18	+ ·2	Sl., Sl.	Good
704	Mar. 19	− ·2	Sl., Sl.	Good
705	Mar. 19	+ ·2	Sl., Sl.	Good
920	Sept. 22	−1·8	Su., M.	Good
937	Oct. 5	− ·7	Su., Sl.	Good
947	Oct. 13	− ·3	Su., M.	Fair
955	Oct. 19	− ·5	Su., Sl.	Good
971	Oct. 27	− ·9	Su., Sl.	Fair
978	Nov. 2	− ·4	Su., Sl.	Good
1135	1913 Feb. 12	− ·2	Su., M.	Good
1146	Mar. 6	− ·5	M., Sl.	Poor
1155	Mar. 8	− ·4	M., Su.	Poor
1156	Mar. 8	·0	M., Su.	Good
1164	Mar. 9	− ·3	Sl., Su.	Good

The first four were taken when the star was east of the sun, the next six about six months later, when it was west of the sun, and the last five again to the east.

The third column gives the hour angle of the star at the time the plate was exposed, plus indicating west of the meridian, and minus east. So far as possible, all observations for parallax are made near the meridian with the telescope west of the pier. In this series one plate has an hour angle greater than one hour, and three, only, greater than half an hour.

Two exposures were made upon each plate, the exposure time varying from 6 to 15 minutes according to the transparency of the atmosphere and the sensitiveness of the plate emulsion. The guiding was done by the observers indicated in column 4: Sl = Slocum, Su = Sullivan, and M = Professor S. A. Mitchell of Columbia University. The plates were measured by the writer, and the computations made by Miss Magill.

Six comparison stars were selected, as symmetrically situated as possible, and as near as possible to the mean brightness of the *Nova*. Table II. gives additional data pertaining to the comparison stars.

TABLE II.

No.	B.D.	X (R.A.)	Y (Dec.)	Dependence.	Diameter.	Approximate Magnitudes. Photographic.	Visual.
					mm		
1	+32° 1431	−257·6	+106·8	+·151	0·21	10·4	9·1
2	+32° 1432	−254·0	− 21·9	·144	·22	9·4	9·2
3		−194·0	−127·7	·144	·16	10 0	9·7
4	+32° 1434	−161·0	+244·7	·166	·26	8·7	8·6
5	+32° 1441	+402·8	−323·9	·183	·28	9·5	8·7
6		+463·8	+122·0	·212	·19	10·5	9·4
Nova		+ 41·88	+ 1·66		·22		5·5 to 9

The third and fourth columns show the co-ordinates of the comparison stars and the *Nova* in units of the scale of the measuring machine. One division of this scale equals 2″·66 approximately. The fifth column gives what Professor Schlesinger calls the "Dependence" of each comparison star.* The figures indicate the relative influence of each star upon the parallax to be found, and they are used to derive from the measures of a plate the residual for the parallax star on that plate. This residual is called the "Solution" (m) in Table III. Column 6 contains the diameters of the star images in fractions of a millimetre. Isochromatic plates were used in connection with a yellow-colour filter, so these diameters will correspond more nearly with the visual than with the photographic magnitudes. Between the first and last observations the *Nova* diminished in light by about 3·5 magnitudes, but the size of its image on the plates was kept practically constant by the use of the rotating disc with adjustable sector. When the star was brightest the sector was opened 10°, thus reducing the star about four magnitudes. For the last plate the occulting disc was removed entirely, for the magnitude of the *Nova* then was very nearly equal to the mean of the magnitudes of the comparison stars.

The last two columns give the approximate magnitudes. For the photographic magnitudes I am indebted to Professor Parkhurst. The visual magnitudes were taken from the B.D. for the four B. D. stars and estimated from the plates for the other two.

The usual method of reduction was followed. One plate was selected as a standard, in this case No. 920, and all of the others were referred to this standard by a linear relation of the form

$$-ax+by+c=x-x_0.$$

A least square reduction gives the residual for the parallax star on each plate. These residuals involve the displacements due to

* *Astrophysical Journal*, xxxiii. p. 161.

parallax and proper motion, plus a constant. Each plate then gives an equation of the form

$$c + T\mu + P\pi = m,$$

from which the parallax and proper motion may be found. Table III. contains the data for the fifteen equations of this form. The unit of T is 100 days, and P is the parallax factor parallel to the equator. All quantities are expressed in terms of scale divisions except where otherwise indicated.

TABLE III.

Reductions for Nova Geminorum (2).

Plate.	Solution (m).	Weight (p).	Parallax Factor (P).	Time in Days (t).	Residual (v).	$\sqrt{p}\cdot v$ in Arc.
702	−0·024	0·7	−·966	−188	+·005	+0″·01
703	−·059	1·0	−·966	−188	−·030	−·08
704	−·025	1·0	−·970	−187	+·004	+·01
705	−·007	1·0	−·970	−187	+·022	+·06
920	−·029	1·0	+·980	0	−·003	−·01
937	−·018	1·0	+·995	+13	+·008	+·02
947	−·022	0·7	+·980	+21	+·004	+·01
955	−·033	1·0	+·956	+27	−·007	−·02
971	−·029	0·7	+·909	+35	−·003	−·01
978	−·022	1·0	+·862	+41	+·004	+·01
1135	−·042	1·0	−·665	+143	−·012	−·03
1146	·000	0·4	−·893	+165	+·031	+·05
1155	−·031	0·4	−·908	+167	·000	·00
1156	−·028	1·0	−·908	+167	+·003	+·01
1164	−·037	1·0	−·915	+168	−·006	−·02

Normal Equations.

$$12{\cdot}9\,c + 0{\cdot}374\,\mu - 1{\cdot}675\,\pi = -0{\cdot}3649$$
$$23{\cdot}238\,\mu + 2{\cdot}622\,\pi = -0{\cdot}0180$$
$$+11{\cdot}079\,\pi = +0{\cdot}0723$$

From which

$$c = -0{\cdot}0280$$
$$\mu = -0{\cdot}0006 = -0''{\cdot}0016 \pm 0''{\cdot}005$$
$$\pi = +0{\cdot}0024 = +0''{\cdot}006 \pm 0''{\cdot}007$$

Probable error corresponding to unit weight, $\pm 0{\cdot}009 = \pm 0''{\cdot}024$.

The relative parallax comes out $+0''{\cdot}006$ with a probable error of $\pm 0''{\cdot}007$. According to Kapteyn's table of average parallaxes, that of the comparison stars may be taken as $0''{\cdot}005$, which would make the absolute parallax of the *Nova* $+0''{\cdot}011$, corresponding to 296 light years.

The proper motion also appears to be very small. The solution of the equations of Table III. gives $-0''{\cdot}0016$ for 100 days, which is equal to $\mu_\alpha = -0^s{\cdot}0003 \pm 0^s{\cdot}001$ per year.

For the proper motion in declination seven plates only were used, three in March 1912, three in March 1913, and the standard plate. The parallax in declination was neglected, as it would enter the value of the proper motion by only about two one-hundredths of its value. From these seven plates I find $\mu_\delta = +0''\cdot036 \pm 0''\cdot016$ per year.

Both of these values are, of course, relative to the weighted mean of the proper motions of the comparison stars.

Yerkes Observatory:
1913 *March* 29.

On the Short Period Variable SW Draconis.

By C. Martin and H. C. Plummer. (Plate 17.)

1. The star SW Draconis, like the neighbouring variable of similar type SU Draconis, was among those found to be variable by Miss Leavitt * from an examination of plates of the Harvard map of the sky. It received the provisional number 45·1907, and its position is

$$1900{\cdot}0 \quad 12^h\ 13^m\ 4^s, \quad +70^\circ\ 4'{\cdot}0.$$

Photographic observations were made with the 15-inch reflector of the Dunsink Observatory between 1912 December 31 and 1913 March 25, during which time sixty-three measurable exposures were secured. The whole of the observations and reductions were made by Mr. Martin.

2. The method adopted at this observatory in order to obtain differential estimates of stellar magnitudes has been fully described, † and the details need not be repeated. Only on one point some comment may be added. It has been stated that the diameters of the images are measured with a micrometer instead of measuring the density with a photometer (p. 166). This is true. But the element of photometric estimation is not altogether absent. The measurer makes a preliminary inspection of the images to be measured, and decides mentally on the degree of density within the image where the settings may be most advantageously and consistently made. He thus measures the diameter of a nucleus at the boundary of which a standard photographic density is judged to obtain. It will be readily understood that the faculty of maintaining such a standard needs experience on the part of the observer, and uniform illumination of the plate during measurement. As daylight is found preferable to an artificial source, certain consequences follow. In order to secure steady conditions of light, the measures must be effected within a reasonably short time. This limits the number of comparison stars which can be profitably employed. And further, if for any reason it is desired to include

* *Harvard Circ.*, 127; *A.N.*, 4181.
† *M.N.*, lxxi. p. 511, lxxiii. p. 166.

an additional object on the plate after a set of measures has been made, it is necessary to repeat the comparisons at the same time, since otherwise the consistency which is essential will be lost. These remarks may serve for a clearer understanding of the characteristic features which distinguish the method employed.

3. In order to determine the photographic magnitudes of the comparison stars, two plates were exposed on the region of the variable and on the χ Persei cluster under equal conditions as regards altitude and time of exposure. For the χ Persei stars we have the determinations of Schwarzschild, and hence the means of finding the magnitude scale for the plates. The results are given in the following table:—

TABLE I.

Star.	B.D.	Gr. Ph.	Plate 1036, m_1.	Plate 1042, m_2.	Diff. m_2-m_1.	Adopted Ph. Mag.
a	+70°·691	+70°·5234	9·32	9·37	+·05	9·34
b	+69·653	+69·4830	9·76	9·82	+·06	9·79
c	+69·655	+69·4887	10·02	10·00	−·02	10·01
d	...	+70·5251	10·05	10·05	·00	10·05
e	+69·656	+69·4879	10·75	10·77	+·02	10·76
f	...	+70·5242	11·07	11·01	−·06	11·04

In this case there is practically no systematic difference between the two sets of magnitudes, but the consistency does perhaps leave a little to be desired. What is important for our purpose is that the adopted magnitudes shall form a consistent series, and there is no doubt that the above values satisfy this condition sufficiently well. Further attention will be given to this point later in the paper (§ 6).

4. Table II. contains the details of the observations. The second column gives the date and G.M.T. of the middle of the exposure. The next column gives the corresponding Julian date with decimal fraction of a day from G.M. noon. By the phase in the fourth column is meant the decimal fraction of a day which has elapsed since the epoch, by which is meant the time in each period corresponding to the approximate time provisionally adopted for the maximum, J.D. 2419780·484. The time of exposure is given in the fifth column. The sixth column contains the concluded photographic magnitude of the variable. The next column shows the deviations of the observations from a curve the nature of which will be explained in § 10. The last column gives the observer's impression of the quality of the plates, formed in the process of measuring them.

5. It is notoriously difficult to judge accurately the epoch of maximum. For the Dunsink observations, as stated, we adopted provisionally,

$$\text{Max.} = \text{J.D. } 2419780{\cdot}484.$$

Elements have been given by Sperra * referred to the epoch when

* *A.N.*, 4407.

the star reaches the visual magnitude 9·66 on the rising phase. Assuming that the maximum follows at an interval $0^d{\cdot}43$, these elements become

$$\text{Max.} = \text{J.D. } 2418086{\cdot}2962 + 0^d{\cdot}56965\ \text{E}.$$

The interval is 1694·1878 days, and assuming that 2974 periods have elapsed, we obtain for the period $0^d{\cdot}569666$. Little weight can be attached to the very slight increase as compared with the earlier determination, and this value of the period is quite accurate enough for the proper combination of observations which are included in the short space of three months. The times given here are true, not heliocentric.

TABLE II.

Serial No.	Date.	G.M.T.	Julian Day.	Phase in Days.	Exposure.	Magn.	Obs. - Curve.	Images.
	1912	h m	2419		min.			
1	Dec. 31	7 11	768·299	·348	35	10·63	+ ·05	Good
	1913							
2	Jan. 1	6 52	769·286	·195	45	10·55	+ ·02	Good
3	1	7 37	769·318	·227	45	10·61	+ ·01	Good
4	1	8 30	769·354	·263	45	10·60	+ ·01	Fair
5	1	9 13	769·384	·293	40	10·60	+ ·02	Fair
6	1	10 5	769·421	·330	45	10·60	·00	Good
7	1	10 50	769·452	·361	45	10·65	− ·04	Fair
8	1	11 45	769·490	·399	45	10·58	+ ·25	Good
9	1	13 35	769·567	·476	45	10·65	+ ·03	Good
10	1	14 21	769·597	·506	45	10·43	+ ·01	Good
11	5	8 13	773·342	·264	45	10·62	− ·02	Bad
12	5	8 59	773·374	·296	45	10·62	− ·01	Bad
13	5	9 54	773·413	·335	45	10·65	− ·02	Bad
14	5	10 40	773·445	·367	45	10·70	− ·10	Bad
15	5	11 35	773·483	·405	45	10·90	− ·07	Fair
16	12	8 10	780·340	·426	45	10·65	+ ·02	Bad
17	12	9 2	780·376	·462	45	10·71	+ ·07	Fair
18	12	9 53	780·412	·498	45	10·47	+ ·03	Good
19	12	10 39	780·445	·531	45	10·14	− ·04	Good
20	12	11 34	780·482	·568	45	9·80	+ ·01	Fair
21	12	12 19	780·514	·030	45	10·00	− ·02	Fair
22	12	13 12	780·550	·066	45	10·02	+ ·01	Fair
23	12	13 58	780·583	·099	45	10·14	+ ·02	Fair
24	12	14 52	780·619	·135	45	10·34	− ·02	Fair
25	12	15 38	780·652	·168	45	10·44	+ ·01	Fair
26	15	11 36	783·483	·151	45	10·46	− ·06	Fair
27	15	12 25	783·518	·186	45	10·43	+ ·09	Good
28	15	13 30	783·563	·231	45	10·62	+ ·01	Good
29	29	11 39	797·486	·482	55	10·71	− ·09	Good
30	29	12 40	797·528	·524	55	10·17	+ ·01	Good
31	29	13 47	797·575	·001	55	9·82	− ·03	Good
32	29	14 50	797·618	·044	55	10·08	+ ·02	Good
33	31	7 20	799·306	·023	25	9·88	·00	Fair

TABLE II.—*continued.*

Serial No.	Date.	G.M.T.	Julian Day.	Phase in Days.	Exposure.	Magn.	Obs.–Curve.	Images.
	1913	h m	2419		min.			
34	Feb. 1	9 28	800·395	·542	55	9·97	+·02	Good
35	1	10 28	800·437	·015	55	9·85	−·06	Good
36	1	11 29	800·479	·057	55	10·12	−·04	Fair
37	1	12 50	800·535	·113	55	10·24	−·02	Good
38	5	10 31	804·438	·028	55	9·91	+·04	Good
39	5	11 30	804·479	·069	55	10·02	−·01	Good
40	5	12 36	804·525	·115	55	10·19	+·03	Fair
41	5	13 35	804·567	·157	52	10·38	+·03	Fair
42	6	9 16	805·386	·406	55	10·85	−·02	Good
43	6	10 19	805·431	·451	55	10·77	−·03	Good
44	6	11 39	805·486	·506	55	10·46	−·02	Good
45	6	12 46	805·532	·552	55	9·94	−·03	Good
46	9	7 28	808·312	·484	30	10·69	−·09	Fair
47	9	8 29	808·354	·526	55	10·17	−·03	Good
48	10	8 37	809·359	·392	55	10·62	+·17	Good
49	10	9 37	809·401	·434	55	10·67	−·02	Fair
50	10	12 43	809·531	·564	55	9·79	+·04	Fair
51	11	7 28	810·312	·205	55	10·63	−·02	Fair
52	11	8 27	810·352	·245	55	10·62	+·01	Fair
53	11	10 3	810·419	·312	55	10·55	+·02	Good
54	11	12 8	810·505	·398	52	10·72	+·11	Good
55	26	11 14	825·468	·550	60	9·95	−·02	Good
56	Mar. 11	9 32	838·397	·377	55	10·64	−·01	Fair
57	11	10 28	838·437	·417	55	10·81	−·04	Good
58	11	12 51	838·536	·516	55	10·30	·00	Good
59	14	8 39	841·361	·492	55	10·53	+·01	Fair
60	14	9 43	841·406	·537	50	10·05	−·01	Fair
61	14	10 43	841·448	·010	50	9·82	−·04	Fair
62	25	9 49	852·409	·147	50	10·34	+·04	Fair
63	25	10 39	852·445	·183	50	10·54	−·03	Fair

6. As explained in our paper on SU Draconis (p. 167), the reduction is made by a graphical process based on a linear relation between m and $D^{2/3}$. The resulting magnitudes for each of the comparison stars on every plate are recorded. If we subtract the corresponding magnitudes adopted in Table I. we have six series of residuals, and to these some examination has been given. The results may be summarised thus:—

Stars	*a*	*b*	*c*	*d*	*e*	*f*
Mean correction	$-^{m}\cdot020$	$+^{m}\cdot002$	$+^{m}\cdot020$	$-^{m}\cdot012$	$-^{m}\cdot014$	$+^{m}\cdot007$
Probable error	± ·022	± ·024	± ·024	± ·033	± ·018	± ·020
Extreme range	·14	·19	·17	·25	·16	·11

The mean corrections to the adopted values seem to support Plate 1036 as against Plate 1042, but in any case they are not sufficiently important to require a fresh reduction. The probable

errors for a single plate have been obtained by a rather summary process, but will serve as an approximate indication of their relative order. The extreme range between the highest and the lowest magnitude found for each star will also serve the same purpose.

7. We come here to a critical question affecting work of this kind. We have to rely on the constancy of the comparison stars within very narrow limits indeed. A large variation betrays itself readily enough, provided that at least three comparison stars are used. But minor variations will have a disturbing effect and may be difficult to detect, because few stars are probably altogether free from them. Two stars which have been rejected as standards of comparison on the ground of suspected variability have been measured on all the plates. These are:

1. Gr. ph. + 70°·5230 2. Gr. ph. + 69°·4889 (B.D. 70°·692),

and their concluded magnitudes are given in Table III. The serial numbers correspond with those assigned in Table II., whence the necessary information as to time and length of exposure can be drawn.

TABLE III.

No.	Star 1.	Star 2.	No.	Star 1.	Star 2.	No.	Star 1.	Star 2.
1	10·98	10·31	22	11·16	10·40	43	10·86	10·40
2	10·96	10·40	23	11·14	10·42	44	10·84	10·46
3	11·06	10·51	24	10·99	10·47	45	10·90	10·57
4	11·17	10·39	25	10·81	10·50	46	10·93	10·35
5	11·17	10·49	26	11·11	10·82	47	10·98	10·58
6	11·08	10·43	27	11·06	10·48	48	11·05	10·45
7	11·13	10·58	28	10·90	10·50	49	10·98	10·43
8	...	10·42	29	11·04	10·40	50	10·98	10·37
9	11·03	10·40	30	11·00	10·47	51	...	10·43
10	11·00	10·45	31	11·06	10·52	52	10·99	10·43
11	10·90	10·45	32	11·04	10·43	53	10·99	10·39
12	11·03	10·55	33	11·07	10·46	54	11·08	10·52
13	10·99	10·43	34	11·07	10·47	55	11·15	10·39
14	11·15	10·40	35	11·04	10·44	56	10·94	10·52
15	11·19	10·45	36	10·93	10·44	57	11·14	10·43
16	10·97	10·47	37	10·93	10·55	58	11·06	10·37
17	10·94	10·35	38	10·98	10·50	59	10·97	10·40
18	10·90	10·37	39	11·09	10·49	60	10·88	10·53
19	10·94	10·32	40	11·07	10·45	61	10·99	10·40
20	10·99	10·40	41	11·03	10·48	62	10·98	10·37
21	10·96	10·58	42	10·96	10·51	63	11·06	10·42

8. If we treat these stars as of constant magnitude, as we have treated the comparison stars, we obtain the results which follow :—

Star	1	2
Mean magnitude	$11^m{\cdot}01$	$10^m{\cdot}45$
Probable error	$\pm$ ·060	$\pm$ ·042
Extreme range	·38	·27

Now the average probable error for a single plate in the case of the comparison stars is $\pm$ ·024, and the average range is ·17. The figures just given are appreciably larger, and we must conclude that the two stars in question are variable. Their periods, which are probably short if not irregular, have not been found. It is just possible that there is a trace of variation in the comparison star *d*. Uncertainties of this kind in the standards of comparison introduce a disturbing element which cannot be altogether avoided. It may be that the region of SW Draconis is particularly unfortunate in this respect. On this point further reference will be made.

9. We now turn to the discussion of SW Draconis itself. The concluded magnitudes were plotted according to the adopted period, and a smooth curve was drawn through them. The ordinary method of periodic analysis was then carried out, based on twenty-four ordinates read off at equal intervals over the period. This process gave the following formula for the magnitude of the star :—

$$\begin{aligned} m = 10{\cdot}42 &+ {\cdot}36 \sin(\theta + 239^\circ) + {\cdot}18 \sin(2\theta + 242^\circ) + {\cdot}07 \sin(3\theta + 307^\circ) \\ &+ {\cdot}05 \sin(4\theta + 302^\circ) + {\cdot}04 \sin(5\theta + 310^\circ) + {\cdot}02 \sin(6\theta + 326^\circ) \\ &+ {\cdot}01 \sin(7\theta + 207^\circ) + {\cdot}03 \sin(8\theta + 221^\circ) + {\cdot}02 \sin(9\theta + 292^\circ) \\ &+ {\cdot}02 \sin(10\theta + 96^\circ) + {\cdot}03 \sin(11\theta + 158^\circ) \end{aligned}$$

where the origin of θ corresponds with the epoch mentioned above (approximately the epoch of maximum) and the period 360° with the period of the star. The slow and irregular convergence of the series is once more evident, even more so than in the case of SU Draconis. It must be admitted that the harmonic representation of the curve is not entirely satisfactory.

10. The curve which is shown in the accompanying fig. 1 (Plate 17) is based on the complete series given above. This is the curve referred to in Table II., and the discordances of the observations from it are shown in the seventh column. Over the greater part of the period the agreement is excellent. It is the absolutely sudden drop at the minimum which an analysis limited to a certain number of terms, as in the present case, cannot follow. Even the attempt to do so has spoilt the representation of the curve in the neighbourhood of the maximum. If the effect is illusory, it must be attributed to a want of constancy in the comparison stars. It may be that the curve is irregular at the minimum. But this is not easy to accept in view of the regularity

throughout the remainder of the period, except perhaps near the maximum. On the other hand, the effect may be real. In that case the present observations will tend to obscure it, and must indeed be difficult to interpret, because each exposure covers about one-sixteenth of the whole curve. For this reason it is scarcely worth while to extend the analysis to terms of shorter period. The result would merely be to secure a more faithful representation in the mathematical sense, without any assured physical basis. It is not implied that in other cases, when the observations do not involve an integrated effect, terms of short period as compared with the whole period may not be profitably studied. Thus the periodicities of S Persei found by Professor Turner (p. 124) seem to be derivable from a main period of 10930 days (or nearly 30 years), which gives the sub-multiples:—

	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{10}$	$\frac{1}{13}$	$\frac{1}{16}$	$\frac{1}{29}$	$\frac{1}{30}$
=	5465	3643	1093	841	683	377	364
Observed	6000?	3669	1073	845	665	376	363

And, similarly, there is some suggestion of a main period of 133·5 years for sun-spots. But that is not the case here. It is not that terms of short period may not exist, but that it is clear that they cannot be properly discussed by our present method. The discordances of the observations are apparently exaggerated by the local imperfections of the curve of reference. Even so their average amount is only $\pm 0^{m}\cdot 036$, and though this is distinctly larger than the corresponding figure ($\pm^{m}\cdot 026$) found in the work on SU Draconis, it is still fairly satisfactory.

11. A very careful study of the star has been made by Sperra (*loc. cit.*). The light curve has been redrawn from the means of normal groups given in his Table VII., and submitted to analysis in the same way as the photographic light curve. The epoch has been reduced to the time of maximum, assuming that this occurs $0^{d}\cdot 43$ after the visual magnitude 9·66 is reached, and by a suitable scale the observer's steps have been converted into magnitude. The following formula is obtained:—

$$\begin{aligned} m - m_0 = {} & \cdot 30 \sin(\theta + 237^\circ) + \cdot 12 \sin(2\theta + 253^\circ) + \cdot 09 \sin(3\theta + 299^\circ) \\ & + \cdot 03 \sin(4\theta + 281^\circ) + \cdot 02 \sin(5\theta + 296^\circ) + \cdot 01 \sin(6\theta + 334^\circ) \\ & + \cdot 01 \sin(7\theta + 354^\circ) + \cdot 01 \sin(9\theta + 198^\circ) + \cdot 01 \sin(10\theta + 207^\circ) \end{aligned}$$

If this is compared with the formula given in § 9, it will be seen that the agreement is excellent for the first six components, both in the relative order of the coefficients and especially in the phases. Beyond this point the correspondence between the two series comes to an end. Perhaps we should be well content to have traced it so far.

12. The following quotation made by Sperra from an earlier paper may be repeated with advantage:—

"The rise requires about four hours, at first slow after

minimum for an hour and a half, then very rapid until maximum, no halt is made and the decline sets in immediately, and for over an hour is as rapid as the rise, a halt is made at two to three hours after maximum, the decline then increases for nearly two hours more, and then for the next five hours is very flat—the lowest point being reached ten hours after maximum. The range is about a magnitude, or 19 light steps on my scale."

If this description is compared with our curve it will be seen that the correspondence is practically perfect. The visual range adopted subsequently appears to be about $0^m{\cdot}8$, while the photographic range is about $1^m{\cdot}1$, a ratio which may be considered quite normal. But when the visual observations were continued, and a more comprehensive discussion was made, two discrepancies made their appearance. These relate to the maxima and minima, both of which, according to Mr. Sperra, vary at different epochs. For the reason already stated in § 10, we are disinclined to admit such a variation unless established by the clearest evidence.

13. The Dunsink observations cover too short an interval of time to decide the question. Indeed, visual observations for over a year failed to detect a single anomalous maximum. On the other hand, Mr. Sperra's evidence for his conclusion is apparently very strong. Is there any way out of the difficulty? We must examine his comparison stars on this point. His chart and Table V. give the following identifications and visual magnitudes:—

Table IV.

Star.	B.D.	Gr. Ph.	Visual mags.
(*c*)	$+70^\circ{\cdot}692$	$+69^\circ{\cdot}4889$	9·32
(*a*)	...	$+70^\circ{\cdot}5251$	9·52
(*d*)	...	$+70^\circ{\cdot}5232$	9·87
(*b*)	...	$+70^\circ{\cdot}5241$	10·19
(*e*)	...	$+70^\circ{\cdot}5230$	10·35

Thus, assuming that the stars have been rightly identified, we find that $(a)=d$, $(c)=2$, and $(e)=1$. As we have seen, there is good reason to believe that both the stars which mark the upper and the lower limit of the visual light scale are variable. Indeed, as regards the brighter of the two stars this is no new discovery, for the probable variability of B.D. $+70^\circ{\cdot}692$ was announced * by Mr. Sperra himself. A series of five observations give a range of $0^m{\cdot}35$ as compared with (*a*), which is greater than that found in § 8. The star received the provisional number 38·1910. But though its variability was thus suspected, its influence on the concluded magnitudes of SW Draconis near the maximum does not seem to have been taken into account. Yet its effect must be eliminated if any certain conclusion as to the form of the curve at this point is to be established. A similar consideration applies

* *A.N.*, 4407.

to the form of the curve near minimum if, as seems clearly possible, the star (*e*) is also variable. The difficulty here is even greater if we admit the very sudden drop in brightness just preceding the minimum, since this feature may be difficult to follow even with the help of fixed standards of comparison. On the whole, we may fairly keep an open mind on the subject of these supposed irregularities at different epochs of maximum and minimum phase, although Mr. Sperra's work is of such an order that one can only differ from his conclusions with diffidence.

14. The question naturally arises whether the occurrence of variables among the stars chosen for comparison is normal or whether it is due to some observable peculiarity in this region of the sky. SW Draconis (like SU Draconis) is a star of cluster type, but it is not a member of a cluster. The following four plates were taken with longer exposures than the ordinary series in order if possible to detect any unusual feature in the vicinity of the star:—

Plate 1091	1913 Feb. 27	Exposure $3^h\ 22^m$
1092	Mar. 3	2 1
1094	6	5 19
1096	7	6 0

The first of these plates revealed a faint nebula running southward from a point on the preceding side of the variable. The second exposure was curtailed by cloud. The last two exposures, which were confirmed by plates exposed simultaneously with the Lewis and Gibson 6-inch doublet, show the nebula quite plainly with a fair amount of detail and some indication of an extension running in the direction of the star 2. The main branch has the shape of a sign of integration, and occupies a characteristic vacant space although there are faint stars embedded in it. The variables are at some distance from the nebula, and it must be uncertain whether it is concerned in their variability. But its existence at least confirms the suspicion that the field is not altogether a normal one. Since the position of the nebula was definitely known, an examination has been made of the whole series of ordinary plates. The exposures are not far short of an hour each, and a faint trace of the image is found on several of them. A curious fact which may be recorded is that the relative intensities of such images, which are of course in all cases very faint, bear apparently no relation to any known circumstances which might affect them. Thus the two-hour exposure on Plate 1092 gives actually a fainter image than has been obtained on another exposure of 50^m, and successive plates on the same night either show an obvious image or none at all without any apparent reason for the difference. As such sudden changes in the brightness of the nebula seem to be out of the question, they must be attributed to the anomalous behaviour of the sensitive film in recording excessively faint objects.

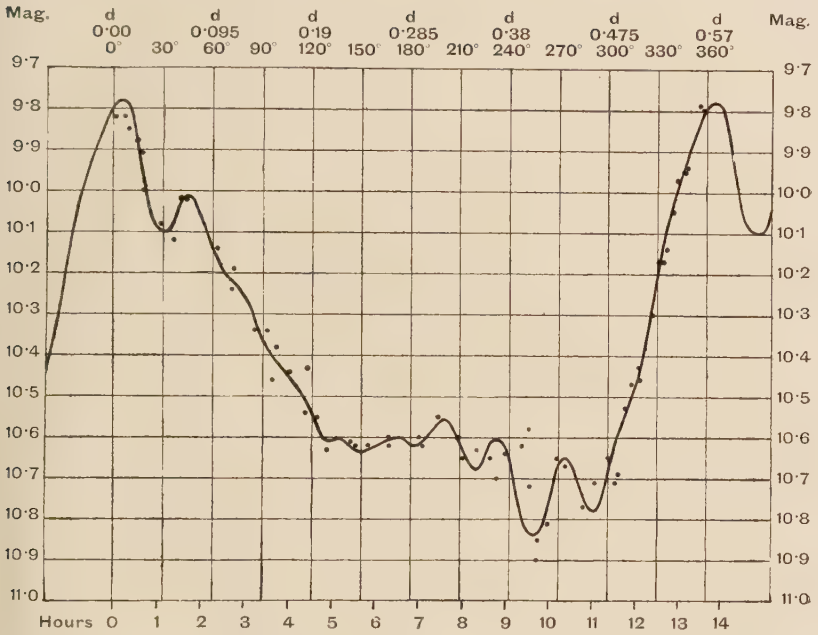


Fig 1.—Photographic Light Curve of SW Draconis.

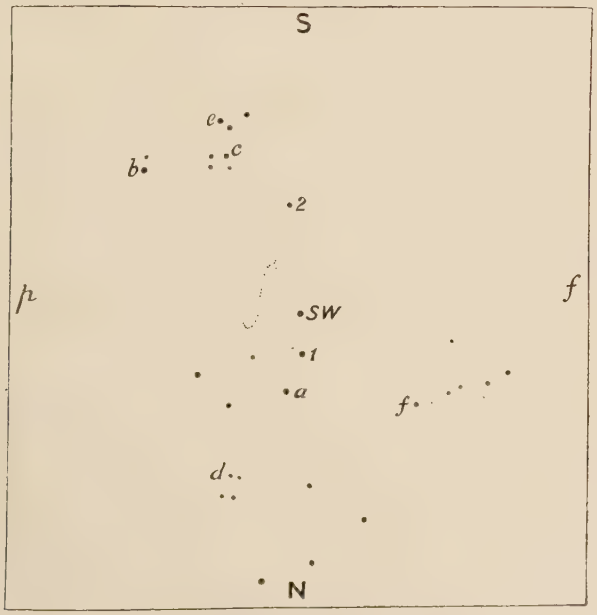


Fig. 2.—Region of SW Draconis.

The position of the nebula, with the photographic comparison stars, is indicated in fig. 2 (Plate 17).

15. The positions of the stars mentioned in this paper, as deduced from the *Greenwich Astrographic Catalogue*, are given in Table V.

TABLE V.

Gr. Ph.	R.A. 1900.	Decl. 1900.	Designation.
	h m s	° ′ ″	
+69°4830	12 8 52·40	+69 42 48·6	*b*
+69·4879	11 0·76	69 36 42·6	*e*
+69·4887	11 2·65	69 41 21·4	*c*
+70·5241	11 4·93	70 16 18·1	(*b*)
+70·5251	11 6·16	70 26 13·7	*d*=(*a*)
+70·5234	12 41·76	70 14 38·4	*a*
+69·4889	12 46·12	69 48 33·6	2=(*c*)
+70·5225	13 4·13	70 3 58·8	SW Draconis
+70·5230	13 10·53	70 9 35·3	1=(*e*)
+70·5242	16 27·45	70 16 47·1	*f*
+70·5232	12 17 25·12	+70 8 8·6	(*d*)

16. SUMMARY.

§ 1. The star and the observations.
§ 2. Remarks on the method employed.
§ 3. The comparison stars (Table I.).
§ 4. Details of the observations of SW Draconis (Table II.).
§ 5. Determination of the period adopted.
§ 6. Result of examining the residuals of the comparison stars.
§ 7. Observed magnitudes of two suspected variables in the same field (Table III.).
§ 8. Evidence of their variability.
§ 9. Periodic analysis of the photographic observations.
§ 10. Discussion of the photographic light curve.
§ 11. Periodic analysis of Sperra's visual light curve.
§ 12. Comparison of general character of photographic and visual light curves.
§ 13. The stars forming the visual light scale (Table IV.). Effect of the probable variability of the two terminal points on the form of the light curve at maximum and minimum.
§ 14. Existence of a nebula in the neighbourhood of the variables.
§ 15. Positions of the eleven stars referred to in the paper (Table V.).

Dunsink Observatory:
1913 *April* 9.

Preliminary Discussion of the Galactic Motions of the Bright Stars of Type I., with some Additional Material. (Stellar Motions, No. 5.) By H. C. Plummer.

1. The work to which the present paper is a further contribution has grown by stages which could not be foreseen at the outset. As a consequence the first three papers of the series were not numbered, but for convenience of reference they may now be placed in the following order:—

No. 1. *Monthly Notices*, lxxii. p. 170.
No. 2. *Lick Obs. Bull.*, No. 212.
No. 3. *Monthly Notices*, lxxii. p. 555.
No. 4. „ „ lxxiii. p. 174.

In No. 2 two misprints have been noticed, and may be corrected here:

P. 32. P.G.C., No. 1022. For $v = -39{\cdot}9$ read $+39{\cdot}9$;
P. 33. „ 2729. For $w = +31{\cdot}1$ „ $+3{\cdot}1$;

and in No. 4:

P. 177. P.G.C., No. 1159. For $w = -{\cdot}02$ read $-0{\cdot}2$;

while in the last line of p. 183, $V = -2$ has dropped out.

2. As explained in No. 2, the radial velocities of the type A stars discussed in that paper were taken from a MS. list most kindly placed at my disposal by Dr. Campbell before publication. Subsequently a list of velocities incorporating the Lick Observatory observations for this class of stars was published in *L.O.B.*, 211. As the published list is based upon later material than was available for the earlier one, the two lists differ considerably. A few stars included in the MS. have been omitted in the published form, while a greater number have been added. Later observations have served to correct the earlier results, in some cases materially. In work which is in the nature of things provisional, it is unnecessary to take into account all the minor corrections which have been introduced. But the larger changes require a fresh calculation, and the stars affected are therefore included in the following Table I., together with the new material. Their P.G.C. numbers are 107, 1220, 1611, 1763, 2407, 2559, 2930, 3101, 3654, 4832, 5084, and 5464. In the case of three stars—P.G.C., Nos. 1478, 3961, and 5582—an error has been found in the earlier computation, and they also have been included in Table I. The remaining 62 stars are additional material, and together with those in the corresponding table of Stellar Motions, No. 2, make a total of 234 stars of type A. In form the table resembles Table I. in Nos. 3 and 4. The numbers in the first column and the proper motions in columns 8 and 9 are taken from the *Preliminary General Catalogue* of Boss. The approximate positions in columns 6 and 7 are referred to the equinox of 1900·0. The type and magnitude m are taken from *L.O.B.*, 211, but follow the *Revised Harvard Photometry*. The radial velocities V are also those given in *L.O.B.*, 211. The parallax ϖ has been derived by the method fully explained in No. 2. The three components of velocity (u, v, w) are relative to the Sun, and expressed like V in kilometres per second. Their

directions are (u) towards the First Point of Aries, (v) towards the point (6^h, 0°), and (w) towards the North Pole. The fifth column under m_0 gives the absolute magnitude at the standard distance corresponding to the parallax 0″·01.

3. In all a total of 459 stars belonging to Secchi's Type I. have now been discussed on the hypothesis of strictly galactic motion. The components of velocity which result have been referred to an equatorial system of co-ordinates. But in order to gain a clearer idea of their systematic character it is desirable to refer them to a galactic system. The following scheme of transformation has been adopted:—

$$P = +\cdot127\,u - \cdot980\,v - \cdot156\,w.$$
$$Q = -\cdot473\,u + \cdot079\,v - \cdot877\,w.$$
$$N = -\cdot872\,u - \cdot185\,v + \cdot454\,w + 6\cdot50.$$

According to this scheme N is the component towards the galactic pole (192°, +27°), corrected for the assumed component of the solar motion, 6·50 km./sec. Hence N = 0 for all those stars which are supposed to satisfy the primitive hypothesis. The component P is directed towards the point (277°·4, −9°), which is a point in the galactic plane not far distant from the apex of Schwarzschild's ellipsoid (273°, −6°). The other component Q is also in the galactic plane and at right angles to P, being directed towards the point (170°·5, −61°·3). Tables II., III., and IV. give the components P and Q for all stars of the types B–B5, B8–B9, and A which apparently conform to the law of galactic motion. For these N = 0. The numbers of a few stars are marked thus * to indicate that the same stars appear also in the following tables V.–VII., the contents of which have yet to be explained.

4. In the course of the discussion of stars of type B–B5 contained in No. 4 a particular group of eleven stars which had yielded negative or abnormal parallaxes were assigned positive parallaxes of an amount according to their types and visual magnitudes suggested by the results which had been drawn from the more favourable stars of their class. The components of velocity thus obtained were distinctly reasonable in character and give a surprisingly good approximation to the solar motion, thus bearing out a remark made quite independently in another connection (*A.N.*, 4622, p. 264). The conditions of this case were particularly favourable, because the stars were situated in the region of 6^h where the proper motions are small and approximate parallaxes suffice. The same method has now been extended to the other unfavourable cases among the stars of the same type, and the results are given in Table V. The parallaxes have been assigned in accordance with the indications of Tables II. and IV. in No. 4. To some extent the choice has been rather arbitrary, and perhaps not entirely consistent. But absolutely no attempt has been made to adjust the values in order to square with the hypothesis of galactic motion. A few stars in Table V. are marked thus †; they are the same as those marked * in Table II.

TABLE I.

P.G.C.	Star.	Type.	m.	m_0.	R.A.	Decl.	μ_α.	μ_δ.	V.	ϖ.	u.	v.	w.
					h m	° ′	s	″		″			
107	– Toucanæ	A2	5·2	6·8	7 3	−63 35	+·0120	−·039	+14	·0205	− 4·5	+ 18·1	− 15·6
253	υ Phœnicis	A3	5·2		15 48	−42 1	+·0025	−·002	− 6	neg.			
300	υ Piscium	A2	4·7	7·0	18 30	+26 44	+·0017	−·012	+10	·0286	+ 7·3	+ 6·4	+ 2·3
463	α ,,	A2p	4·3	7·3	29 13	+ 2 17	+·0028	−·006	+10	·0394	+ 5·4	+ 8·8	− 0·4
482	β Trianguli	A5	3·1		30 54	+34 31	+·0123	−·046	− 2	neg.			
550	ι Cassiopeiæ	A5	4·6		35 12	+66 57	−·0006	+·014	+ 2	,,			
880	28 Eridani	A2	5·0	6·3	55 50	−24 11	+·0032	+·054	+21	·0186	+ 4·2	+ 26·0	+ 4·4
986	b^1 Persei	A2	4·6	(0·4)	62 41	+50 3	+·0056	−·054	+23·1	(·0015)	(−87·1)	(+211·8)	(− 95·3)
1012	– Eridani	A	5·3		64 4	−20 53	+·0031	−·005	− 9	neg.			
1130	κ Doradus	A5	5·4	(−0·7)	70 43	−59 55	+·0007	+·037	−18	(·0006)	(+40·3)	(+238·5)	(+160·1)
1161	7 Camelop.	A2	4·4	3·2	72 19	+53 36	−·0016	+·007	− 8·9	·0058	+ 8·0	− 13·6	− 4·6
1220	β Eridani	A2	2·9	6·3	75 44	− 5 13	−·0059	−·079	− 8	·0470	+ 6·2	− 11·6	− 7·1
1363	θ^1 Orionis	A	5·4		82 35	− 5 27	+·0002	+·004	+19	neg.			
1478	β Aurigæ	Ap	2·1	6·1	88 3	+44 56	−·0044	−·005	−18·1	·0618	+ 3·1	− 13·4	− 13 8
1611	8 Monocerotis	A5	4·5	4·8	94 37	+ 4 39	−·0008	−·003	+21	·0115	+ 3·3	+ 20·4	+ 0·4
1763	θ Geminorum	A2	3·6	3·3	101 33	+34 5	+·0005	−·054	+ 8	·0086	− 7·8	+ 21·3	− 20·7
1886	λ ,,	A2	3·6	7·8	108 5	+16 43	−·0033	−·048	− 9	·0681	+ 5·8	− 7·2	− 6·1
1952	ρ ,,	A5	4·2		110 40	+31 59	+·0117	+·183	− 4	neg.			

1956	64 Geminorum	A2	5·0	5·8	110 47	+28 19	−·0024	−·059	+28	·0145	− 2·0	+ 34·4	− 4·1
2078	ϕ ,,	A2	5·0		116 51	+27 1	−·0022	−·037	−22	neg.			
2237	30 Monocerotis	A	4·0	6·3	125 10	− 3 35	·0044	·025	+14	·0292	+ 1·4	+ 16·5	− 4·9
2327	γ Cancri	A	4·7	6·2	129 23	+21 50	·0073	−·050	+29	·0203	− 0·9	+ 38·5	0·4
2407	α ,,	A5	4·3	6·7	133 15	+12 15	+·0225	−·039	−15	·0301	+ 5·6	− 14·4	− 9·4
2426	ν ,,	A	5·4		134 13	+24 51	·0000	−·005	−14	neg.			
2476	18 Ursæ Maj.	A5	4·9	6·8	137 15	+54 26	+·0070	+·060	−19	·0243	+ 7·4	− 23·1	− 9·5
2495	38 Lyncis	A	3·8	4·4	138 9	+37 14	−·0021	−·135	+ 4	·0132	−17·6	+ 27·9	− 36·8
2559	32 Hydræ	A3	4·5	4·8	141 43	− 0 45	−·0012	−·019	+ 5	·0114	+ 1·6	+ 8·3	− 7·9
2723	q Velorum	A2	4·1	6·8	152 38	−41 38	−·0140	+·031	+12	·0343	+ 0·1	+ 24·3	− 4·1
2768	30 Leonis Min.	A5	4·8	4·6	155 3	+34 18	−·0054	−·083	+14	·0091	−17·1	+ 46·3	− 28·3
2809	K Carinæ	A2	4·9	5·9	156 57	−71 29	+·0045	−·033	+ 8·8	·0159	+ 3·8	− 8·6	− 10·5
2928	59 Leonis	A5	5·1		163 53	+ 6 38	−·0036	−·030	−16	neg.			
2930	β Ursæ Maj.	A	2·4	4·6	163 57	+56 55	+·0102	+·028	−12	·0277	+ 6·7	− 16·8	− 8·3
2964	β Crateris	A2	4·5	7·3	166 41	−22 17	+·0001	−·101	+ 6	·0368	+ 0·3	− 0·2	− 13·9
2972	δ Leonis	A2	2·6	3·6	167 12	+21 4	+·0106	−·145	−18	·0157	− 8·0	− 44·2	− 47·8
3005	γ Crateris	A5	4·1	5·2	169 58	−17 8	−·0075	−·003	+ 8	·0162	− 0·9	+ 32·1	− 2·9
3101	β Leonis	A2	2·2	7·3	175 59	+15 8	−·0342	−·123	+1·3	·1036	− 0·2	+ 22·7	− 5·4
3117	γ Ursæ Maj.	A	2·5	5·7	177 9	+54 15	+·0107	+·003	− 9	·0430	+ 5·6	− 10·6	− 7·9
3190	δ ,,	A2	3·4	4·7	182 37	+57 35	+·0137	+·003	−11	·0183	+ 8·4	− 28·2	− 9·7
3192	6 Comæ Ber.	A2	5·1	4·6	182 44	+15 27	−·0050	−·033	+ 5	·0078	−11·3	+ 43·4	− 18·2
3283	23 ,,	A	4·8		187 28	+23 11	−·0052	+·007	−20	neg.			

TABLE I.—*continued.*

P.G.C.	Star.	Type.	m.	m_0.	R.A.	Decl.	μ_α.	μ_δ.	V.	ϖ.	u.	v.	w.
					h m	° ′	s	″		″			
3447	20 Canum Ven.	A5	4·7		198 16	+41 6	−·0111	+·004	+9·5	neg.			
3480	80 Ursæ Maj.	A5	4·0		200 18	+55 31	+·0143	−·024	−12	,,			
3486	K Centauri	A	5·3	9·8	200 49	−50 39	−·0023	−·019	·26	·0812	+16·4	+7·6	+20·2
3506	78 Virginis	A*p*	4·9	13·7	202 16	+4 10	+·0028	−·031	−6	·5835	+6·6	+2·3	−0·8
3512	24 Canum Ven.	A3	4·6	3·3	202 36	+49 32	−·0120	+·015	−19	·0055	−17·8	+102·3	−6·8
3518	25 ,,	A	4·9	11·9	203 15	+36 48	−·0086	+·014	−6	·2482	+4·5	+4·1	−4·0
3612	τ Virginis	A2	4·3		209 8	+2 2	+·0013	−·025	−10	neg.			
3654	κ Bootis	A5	4·6		212 29	+52 15	+·0071	−·015	−19	,,			
3667	ι ,,	A5	4·8	4·0	213 9	+51 50	−·0157	+·086	−18	·0070	−5·9	+114·5	+21·3
3814	16 Libræ	A2	4·6	4·3	222 59	−3 56	−·0067	−·164	+20	·0086	−47·1	+31·9	−91·8
3883	χ Bootis	A	5·3	6·8	227 35	+29 32	−·0058	+·022	−18	·0195	−0·6	+26·6	−4·7
3926	μ Bootis	A3	4·5	12·3	230 11	+37 44	−·0126	+·078	−8	·3701	+3·5	+7·2	−4·7
3928	γ Ursæ Min.	A2	3·1		230 13	+72 11	−·0026	+·012	−8	neg.			
3961	α Coronæ Bor.	A	2·3	6·3	232 34	+27 3	+·0090	−·102	+0·4	·0628	+5·4	−7·9	−7·1
4269	45 Herculis	A*p*	5·3		250 43	+5 26	−·0016	−·045	−17	neg.			
4391	69 ,,	A	4·8		258 34	+37 24	−·0034	+·053	−4	,,			
4454	π Aræ	A	5·3	7·6	262 28	−54 26	−·0027	−·151	−13	·0291	−0·1	+28·4	−2·9
4462	ξ Serpentis	A5	3·6		262 58	−15 20	−·0030	−·071	−42·8	(·0001)			
4475	*o* ,,	A2	4·4		263 57	−12 49	−·0048	−·056	−33	neg.			

4556	95 Herculis	A	5·1		269 19	+21 36	+·0010	+·027	−27	neg.				
4802	θ^1 Serpentis	A5	4·5	6·4	282 49	+ 4 4	+·0031	+·027	−38	·0240	+ 0·2	+ 40·3	+	2·5
4832	ζ Sagittarii	A2	2·7		284 4	−30 1	−·0016	·000	+22	neg.				
5084	ϵ Pavonis	A	4·1	6·7	297 15	−73 10	+·0162	−·132	0	·0325	+ 0·5	+ 21·3	−	4·6
5086	13 Vulpeculæ	A	4·5	3·6	297 18	+23 49	+·0021	+·028	−28	·0067	+ 2·3	+ 40·1	+	6·4
5188	29 Cygni	A	5·0	5·6	302 42	+36 30	+·0054	+·068	−22	·0130	+ 2·1	+ 40·8	+	6·3
5332	ι Microscopii	A5	5·1	7·7	310 26	−44 21	+·0156	−·097	−19	·0335	+ 2·5	+ 33·5	+	4·2
5464	ϵ ,,	A	4·8	8·5	317 58	−32 35	+·0045	−·034	+ 5	·0560	+ 4·6	+ 2·4		4·6
5473	θ ,,	A2*p*	4·9	9·3	318 35	−41 14	+·0075	·000	+ 5	·0774	+ 5·7	+ 1·9	−	2·6
5551	ξ Aquarii	A5	4·8	6·3	323 6	− 8 18	+·0075	−·024	−20	·0198	− 1·2	+ 34·3	−	2·7
5582	ι Piscis Austr.	A	4·4	7·8	324 45	−33 29	+·0029	−·086	+ 6·0	·0483	+ 1·7	+ 3·2	−	9·8
5617	14 Pegasi	A	5·0	5·5	326 21	+29 43	+·0020	−·027	−20	·0128	− 5·7	+ 15·4	−	19·1
5652	η Piscis Austr.	A	5·4	2·6	328 46	−28 56	+·0011	+·002	−10	·0027	+ 6·4	+ 25·8	+	8·4
5695	− ,,	A5	5·5		331 1	−34 30	−·0029	+·021	−30	neg.				
5726	λ ,,	A	5·4	4·9	332 10	−28 16	+·0018	−·003	− 1·3	·0078	+ 4·2	+ 14·1	−	0·5
5904	δ Aquarii	A2	3·5	2·6	342 20	−16 21	−·0033	−·021	+22	·0066	+ 4·8	− 37·4	−	20·4
6005	τ Pegasi	A5	4·6	3·5	348 55	+23 12	+·0020	−·021	−22	·0061	−10·2	+ 24·0	−	24·2
6062	ι Phœnicis	A2*p*	4·8		352 25	−43 10	+·0023	−·009	+26	neg.				

TABLE II.

P.G.C.	Type.	P.	Q.
27	B2	+ 1·6	+ 4·4
122	B2	− 12·8	+ 4·5
123	B3	− 10·2	+ 0·1
175	B3	− 11·2	+36·0
199*	B*p*	−118·1	+29·9
384	B*p*	− 16·4	+11·8
419	B5	− 28·2	+22·6
563	B5	− 13·9	+40·3
780	B3	− 57·1	+36·0
839	B2	− 29·4	+35·7
844	B1	− 12·0	− 5·2
852	B5	− 9·4	− 0·7
856	B5	− 2·8	+11·7
860	B5	− 12·7	+17·7
865	B5	− 7·2	+12·3
869	B5	− 9·8	− 0·9
894	B1	− 16·9	− 2·5
920	B3	− 3·1	+ 0·9
1003	B3	− 18·7	+20·3
1079	B2	− 7·1	+ 3·0
1159	B3	− 11·7	− 1·0
1204	B3	− 13·1	+19·8
1262	B5	− 11·6	+ 1·0
1277	B2	− 15·1	+ 1·3
1301	B1	− 30·4	+ 2·6
1303	B2	− 12·1	+ 0·9
1314	B2	− 5·9	+ 3·7
1339	B	− 16·8	+25·7
1340	B	− 9·3	+11·3
1361	B1	− 23·6	+ 0·9
1370	B	− 18·7	+ 1·7
1375	B3	− 13·7	+ 6·9
1396	B3	− 31·5	+18·1
1467	B5	− 20·3	+ 5·3
1761	B2*p*	− 3·4	− 1·4
1819	B5	− 18·1	+16·5
1845	B3	− 10·7	+ 8·2
1877	B3*p*	− 17·5	+16·1
1910	B3	− 7·2	+10·1
1934	B5*p*	− 29·1	+19·9
2017	B5	− 13·2	+13·8
2070	B	− 7·6	+13·0
2111	B3	− 10·8	+10·1
2226	B2	− 8·8	+10·8
2266	B5	+ 1·2	− 5·4
2325	B3	− 20·6	+ 5·1
2341	B5	− 9·5	+11·3
2342	B2	− 21·9	− 0·1
2428	B5	− 19·3	+21·0
2457	B3*p*	− 8·2	+19·5
2467	B5	− 20·0	+24·4
2473	B3	− 7·8	+16·5
2477	B3	− 7·5	+ 8·9
2526	B3	− 8·2	+15·1
2581*	B5	− 67·9	+ 1·3
2600	B3	− 20·1	+15·7
2674	B5	− 7·1	+ 8·3
2755	B5*p*	− 10·8	+ 2·6
2792	B5	− 17·2	+11·2
2811	B5*p*	− 22·5	+16·8
2860	B3*p*	+ 3·3	− 4·5
2862*	B	− 54 8	+ 7·6
2889	B3	− 0·3	+16·1
3162	B5	− 24·9	+14·6
3176	B3	− 13·5	+25·8
3187	B3	− 11·0	+22·0
3200	B3	− 9·3	+16·2
3237	B1	− 5·8	+ 3·0
3289	B3	− 6 9	+ 9·4
3324	B3	− 5·1	+15·5
3328	B1	− 9·9	+10·5
3358	B3	− 17·2	+12·5
3359	B3*p*	− 7·9	+ 4·5
3476	B2	− 9·5	+ 7·4
3521	B1	− 10·1	+ 5·6
3564	B2	− 14·7	+ 9·9
3565	B2*p*	− 15·1	+17·4
3566	B3	− 10·8	+ 9·4
3577	B5	− 18·0	+23·7
3586	B5	− 9·4	+12·7
3602	B3	− 15·6	+ 9·8
3603	B3	− 10·6	+ 5·6
3615	B1	− 6·6	+10·2
3621	B3	− 4·6	+13·7
3670	B5	− 15·2	+ 5·3
3688	B5	− 7·9	+12·5
3699	B3	− 9 8	−17·9
3724	B3*p*	− 12·5	+ 3·8
3745	B2	− 9·8	+10·7
3783	B5	− 16·9	+15·5
3815	B2*p*	− 15·3	+ 6·2
3818	B3	− 6·2	+13·0
3892	B4	− 14·0	+34·2
3905	B3	− 10·6	+21·2
3954	B3	− 4·4	+ 5·2
3991	B5	− 27·7	+41·8

4052	B3	8·4	+11·5
4076	B5	− 21·2	+24·2
4156	B5	12·1	+ 5·7
4162	B5	37·9	6·2
4180	B4	35·9	+36·7
4218	B	− 8·1	+ 9·9
4249	B3	− 10·1	+ 0·5
4281	B2	− 40·3	+49·4
4287	B1*p*	− 26·7	− 17·5
4288	B	− 25·6	− 4·4
4334	B1*p*	+ 7·5	− 3·4
4368	B5	− 18·5	+14·4
4388	B3	− 17·0	+22·5
4399	B3	− 8·1	+ 6·9
4429	B3	+ 4·8	+15·5
4431	B3*p*	25·9	+23·0
4439	B2	8·1	+ 9·7
4479	B3	11·7	+16·8
4548	B5*p*	− 6·5	+ 7·9
4565*	B1	− 64·5	+71·8
4590	B2	− 12·5	+ 9·9
4604	B5	− 12·2	− 0·2
4612	B	− 12·0	+ 4·4
4657	B3	26·0	+25·5
4734	B3	− 11·3	+13·6
4776	B2*p*	− 6·7	+ 9·4
4784	B3	− 10·1	+10·7
4792	B5	− 1·3	+ 1·4
4897	B3	2·7	+14·6
4906	B5	4·0	+15·1
4942	B5	3·7	+22·5
4949	B2	− 26·4	+ 3·0
5004	B5	− 18·2	+61·9
5265	B3	− 21·2	+29·9
5272	B5	− 19·1	+25·7
5361	B2	− 2·6	+ 8·1
5375	B3	− 19·8	+13·3
5469*	B5	+ 62·7	+49·1
5471	B3*p*	+ 3·1	− 4·3
5532	B1	− 12·2	+ 7·3
5563	B2	15·3	+14·0
5609	B3	− 9·5	+22·6
5763	B5	− 2·1	− 3·1
5764	B5	− 16·8	+11·0
5779	B5	+ 18·0	+36·5
5933	B3	− 9·9	+22·0
5939	B5	− 9·1	+ 7·6
6155	B2	− 10·0	+ 9·9
6173*	B5	− 67·4	+10·1

TABLE III.

P.G.C.	Type.	P.	Q.
257	B8	− 7·7	+ 9·1
369	B8	− 4·6	+ 7·8
708	B8	−50·1	+34·5
877	B8*p*	−10·5	− 3·0
1001	B9	−13·6	+ 9·9
1250	B8*p*	−20·2	+ 7·8
1304	B8	14·1	+ 7·7
1438	B9	−19·6	− 0·4
1639	B9	−18·7	+ 2·8
1924	B8	−13·6	+ 8·1
2445	B8	−33·5	+11·5
2604	B9	−21·8	+20·6
2790	B9	−26·3	+20·3
3048	B8	−22·1	+11·4
3054	B9	− 6·1	+10·1
3191	B8	− 8·3	− 5·2
3300	B8	− 9·8	+17·4
3305	B8	− 9·6	+20·7
4089	B9	+27·1	+ 5·5
4094	B9	−11·1	+13·0
4498	B8	− 19·4	+ 1·0
4637	B8	− 15·3	− 0·2
4796	B9	+ 1·8	−14·9
4929	B8	− 27·9	+32·1
4999	B9	− 21·3	− 3·7
5199*	B9	−127·4	+15·3
5288	B9	− 11·6	+ 9·7
5310	B8	− 24·6	+10·0
5349*	B8	− 29·4	+98·3
6054	B9	− 14·9	− 1·5

TABLE IV.

P.G.C.	Type.	P.	Q.
10	A	- 10·0	+20·9
43	A	+ 20·0	- 3·4
101	A2	- 5·0	+11·2
107	A2	- 15·9	+17·2
148	A	+ 25·5	-34·6
300	A2	- 5·8	- 5·0
422–3	A*p*	- 8·2	+14·3
428	A5	- 10·8	+15·3
441	A5	+ 15·3	- 2·8
446	A2	+ 36·4	- 7·0
463	A2*p*	- 7·8	- 1·5
474	A*p*	- 43·0	+ 7·7
629	A5	- 41·9	+ 2·6
630	A	- 2·0	+ 5·9
642	A2	- 1·9	+ 6·1
757	A2	+ 5·8	- 3·9
791	A2	+ 4·6	- 8·4
880	A2	25·7	- 3·8
932	A	+ 4·1	+28·5
989	A5	- 11·3	- 5·5
1022	A2	- 39·7	- 3·0
1026	A3	- 43·0	- 4·9
1029	A	- 36·4	- 6·2
1081	A*p*	- 15·1	+23·0
1161	A2	+ 15·0	- 0·9
1194	A5	- 40·4	- 8·4
1203	A	- 18·5	+20·8
1220	A2	+ 13·3	+ 2·4
1241	A*p*	- 23·4	+18·1
1281	A	- 31·5	+ 6·9
1453	A2	+ 34·6	-13·0
1478	A*p*	+ 15·7	+ 9·5
1481	A5	-101·7	-33·5
1611	A5	- 19·7	- 0·4
1657	A	- 10·6	+32·9
1682	A	- 30·0	+14·9
1716	A2	- 3·7	- 9·0
1732	A	+ 11·6	+ 2·4
1763	A2	18·7	+23·6
1874	A*p*	+ 9·1	+ 4·5
1886	A2	+ 8·8	+ 2·0
1928	A	- 54·1	+25·4
1956	A2	- 33·4	+ 7·2
1979	A	- 4·8	+ 0·7
2009	A	- 25·9	+20·1
2035	A2*p*	- 19·7	+24·3
2108	A2	- 31·7	+ 8·1
2181	A3	- 5·8	+17·6
2207	A5	- 18·5	- 5·8
2237	A	- 15·2	+ 4·9
2307	A5	- 11·6	+16·3
2327	A	- 37·7	+ 3·8
2329	A3	- 12·2	+16·7
2358	A	- 10·8	+20·9
2361	A	- 13·6	+17·5
2399	A	+ 33·2	+33·3
2404	A5	- 14·6	+ 0·6
2407	A5	+ 16·3	+ 4·5
2440	A5	+ 4·2	+ 4·6
2443	A3*p*	+ 5·5	+ 0·7
2476	A5	+ 25·0	+ 3·0
2493	A	- 31·0	-20·9
2495	A	- 23·8	+42·8
2559	A3	- 6·7	+ 6·8
2640	A	- 2·1	- 4·3
2690	A	- 52·4	- 5·3
2694	A*p*	- 3·2	+21·9
2697	A	- 17·6	+ 7·7
2723	A2	- 23·2	+ 5·5
2729	A	- 39·5	+ 0·2
2768	A5	- 43·2	+36·6
2809	A2	+ 10·5	+ 6·7
2930	A	+ 18·7	+ 2·8
2961	A	- 16·7	-26·8
2964	A2	+ 2·4	+12·1
2972	A2	+ 49·8	+42·2
2974	A	- 18·6	+26·4
3005	A5	- 31·1	+ 5·4
3036	A2*p*	- 4·0	-14·7
3068	A	- 28·7	+ 3·1

3101	A2	21·4	+ 6·6
3117	A	+ 12·3	+ 3·5
3174	A2	- 8·0	- 1·5
3190	A2	+ 30·2	+ 2·3
3192	A2	- 41·1	+24·7
3210	A	- 22·3	+11·8
3220	A5	- 3·9	-28·3
3302	A	- 38·0	- 1·3
3352	A5	+ 9·9	- 2·3
3363*	A*p*	+ 76·0	- 2·0
3371*	A*p*	-147·6	+ 1·2
3384	A	- 8·8	-11·3
3409	A	- 10·4	+13·8
3452	A2	- 47·4	+18·4
3486	A	- 8·5	-24·9
3506	A*p*	- 1·4	- 2·2
3512*	A3	-101·5	+22·5
3518	A	- 2·8	+ 1·7
3626*	A	- 64·7	+17·7
3633	A1	12·6	8·9
3667*	A5	-116·2	6·9
3712	A2	- 8·7	+14·7
3749	A	+ 8·2	- 5·6
3814*	A2	- 23·0	+105·3
3825*	A	- 79·1	+15·1
3880	A3	- 7·4	+15·2
3883	A	- 25·5	+ 6·5
3916	A	- 14·9	- 3·0
3926	A3	- 6·0	+ 3·0
3949	A2	- 6·7	+ 9·7
3961	A	+ 9·5	+ 3·0
4112	A	- 21·3	+ 5·6
4119	A2	- 8·2	+ 0·4
4143	A	- 13·0	- 0·8
4155	A3	- 9·7	+ 6·3
4182	A*p*	- 4·9	+ 1·8
4189	A2	- 30·4	- 4·0
4284	A2*p*	+ 24·3	+14·9
4328	A	- 24·9	+11·8
4346	A3	- 2·1	- 0·1
4395	A	+ 9·9	- 4·0
4419	A	- 10·4	+ 9·0
4454	A	- 27·3	+ 4·7
4503	A2*p*	20·5	- 4·1
4566	A5	- 35·5	- 3·0
4645	A	- 35·4	+47·1
4722	A	- 13·8	+ 5·1
4725	A2	- 26·3	+16·6
4802	A5	- 39·9	+ 0·9
4824	A	+ 1·7	+33·0
4843	A	- 29·0	-14·1
5050	A2	+ 4·9	- 0·5
5062	A5	- 42·6	- 7·9
5081	A	- 9·8	+ 6·2
5084	A	- 20·1	+ 5·5
5086	A	- 40·0	- 3·5
5110	A3	- 20·7	+ 1·8
5132	A5	- 20·8	+11·3
5171	A	- 34·3	0·0
5188	A	- 40·7	- 3·3
5214	A	- 4·7	+ 8·5
5282	A	- 26·1	+ 5·5
5301	A	- 16·9	+ 5·9
5320*	A2	+ 4·8	+ 8·3
5323	A2	+ 19·9	+16·4
5332	A5	- 33·2	- 2·3
5337	A	- 19·7	+12·9
5417	A3	+ 26·9	+19·5
5427	A	- 41·9	+16·0
5464	A	- 1·1	+ 2·0
5467	A5	- 34·9	+ 0·8
5473	A2*p*	- 0·8	- 0·2
5551	A5	- 33·4	+ 5·7
5582	A	- 1·4	+ 8·1
5617	A	- 12·8	+20·7
5624	A	- 14·6	- 1·4
5652	A	- 25·8	- 8·4
5679	A8	- 22·6	+ 2·1
5726	A	- 13·2	- 0·5
5742	A5	- 11·9	- 1·1
5808	A	+ 3·1	+ 3·0
5837	A5	+ 10·6	- 10·6
5858	A	+ 5·0	- 3·7
5893	A	+ 24·9	+25·5
5904	A2	+ 40·5	+12·6
5910	A	- 40·2	- 3·3
5916	A3	- 0·5	+ 1·6
6005	A5	- 21·0	+27·9
6031	A2	- 11·1	+13·8
6068	A2	- 14·4	+ 7·2
6084	A5	+ 4·4	- 1·5

TABLE V.

P.G.C.	Type.	*m.*	ϖ (assumed).	*u.*	*v.*	*w.*	P.	Q.	N.
			″						
199†	B*p*	2·2	·0151	− 5·3	+ 8·4	− 7·0	− 7·8	+ 9·3	+ 6·3
212	B5	4·4	·0100	+ 4·7	+ 4·9	− 2·7	− 3·8	+ 0·6	+ 0·3
604	B2	4·0	·0077	+ 0·1	+ 8·4	+ 0·6	− 8·3	+ 0·2	+ 5·1
627	B5	4·4	·0098	+ 7·1	+ 1·4	− 8·6	+ 0·8	+ 4·2	− 3·9
679	B5	4·7	·0085	+ 3·6	+10·3	− 1·9	− 9·3	+ 0·8	+ 0·6
874	B3	5·0	·0092	− 6·9	+21·8	− 8·7	−20·9	+12·6	+ 4·5
981	B5	4·3	·0107	− 3·6	+18·4	− 6·7	−17·5	+ 9·1	+ 3·2
1066	B3	5·2	·0088	+ 2·9	+ 2·5	−12·6	− 0·1	+ 9·9	− 2·2
1139	B	4·4	·0090	− 3·5	+ 0·3	+ 4·0	− 1·3	− 1·8	+11·3
1147	B3	3·8	·0138	+ 7·4	+17·2	− 0·3	−16·0	− 1·8	− 3·3
1284	B3	4·6	·0108	+ 6·3	+22·6	− 1·5	−21·1	+ 0·1	− 3·9
1315	B3	4·8	·0096	− 3·2	+14·1	− 1·6	−14·0	+ 4·0	+ 6·0
1398	B	2·0	·0120	− 2·0	+13·0	− 3·6	−12·4	+ 5·1	+ 4·2
1399	B3	5·0	·0092	+ 8·4	+20·5	− 4·6	−18·3	+ 1·6	− 6·8
1435	B	2·2	·0120	0·0	+15·8	− 4·7	−14·8	+ 5·3	+ 1·5
1469	B5	5·2	·0090	+13·1	+10·0	−10·0	− 6·5	+ 3·4	−11·3
1490	B3	4·4	·0120	+ 1·9	+15·4	− 9·9	−13·4	+ 9·0	− 2·5
1507	B2	4·7	·0065	− 6·0	+18·6	− 3·3	−18·5	+ 7·2	+ 6·8
1601	B3	3·1	·0168	− 3·4	+17·0	− 9·4	−15·6	+11·1	+ 2·0
1609	B1	2·0	·0100	+ 1·2	+27·8	− 8·9	−25·6	+ 9·4	− 3·7
1634	B3	5·0	·0100	− 1·0	+23·8	+ 4·7	−24·1	− 1·7	+ 5·1
1641	B5	4·5	·0115	− 1·1	+16·9	−10·6	−15·0	+11·1	− 0·4
1660	B1	4·4	·0075	− 8·0	+22·7	− 9·5	−21·7	+13·9	+ 5·0
1793	B5	4·4	·0115	− 7·3	+34·3	− 5·9	−33·6	+11·4	+ 3·8
1804	B1	1·6	·0200	− 5·4	+21·0	−12·2	−19·4	+15·0	+ 1·8
1817	B5	4·1	·0168	− 8·3	+39·1	−19·4	−36·4	+24·0	− 2·3
2581†	B5	4·2	·0109	− 6·6	+12·6	−12·1	−11·2	+14·7	+ 4·5
2862†	B	3·0	·0075	− 4·7	+16·4	− 7·6	−15·5	+10·2	+ 4·1
3716	B2	4·6	·0062	−14·7	+38·4	− 2·3	−39·1	+12·0	+11·2
4093	B2	4·1	·0075	+ 7·3	+21·0	−14·2	−17·5	+10·7	−10·2
4190	B1*p*	5·5	·0058	− 1·9	+26·9	+12·8	−28·6	− 8·2	+ 9·0
4565†	B1	3·9	·0078	− 7·0	+14·3	− 9·5	−13·4	+12·7	+ 5·7
4992	B3	4·8	·0096	− 9·0	+19·1	−16·1	−17·3	+19·9	+ 3·5
5102	B3	4·9	·0094	−15·8	+23·4	−28·6	−20·4	+34·4	+ 3·0
5190	B3	4·8	·0096	− 8·1	+10·1	− 7·2	− 9·8	+10·9	+ 8·4
5223	B3	2·1	·0294	− 7·7	+12·6	− 6·3	−12·3	+10·1	+ 8·0
5469†	B5	4·3	·0100	− 6·9	+ 3·2	− 9·6	− 2·5	+12·0	+ 7·5
5762	B3*p*	4·9	·0104	+ 2·5	+ 1·0	+ 2·7	− 1·1	− 3·5	+ 5·4
5955	B	4·9	·0044	− 6·0	+12·7	− 2·4	−12·8	+ 5·9	+ 8·3
6046	B3	4·9	·0100	−13·6	+13·9	−13·3	−13·2	+19·2	+ 9·8
6173†	B5	5·0	·0080	− 6·2	+15·4	+ 2·4	−16·3	+ 2·0	+10·2

TABLE VI.

P.G.C.	Type.	m.	ϖ (assumed).	u.	v.	w.	P.	Q.	N.
			"						
100	B9	4·5	·0257	−11·2	+14·9	− 3·5	−15·5	+ 9·6	+11·9
433	B9	5·0	·0204	+ 7·8	− 6·9	−13·6	+ 9·9	+ 7·7	− 5·2
580	B9	5·0	·0204	+ 8·5	− 0·1	− 8·4	+ 2·5	+ 3·4	− 4·7
621	B9	4·3	·0282	− 6·6	+14·4	− 3·2	−14·4	+ 7·0	+ 8·1
781	B9	4·4	·0269	− 2·4	− 1·8	− 5·3	+ 2·3	+ 5·6	+ 6·5
786	B9	4·8	·0224	− 4·1	− 1·3	− 6·0	+ 1·7	+ 7·1	+ 7·6
885	B8	4·9	·0093	−29·6	+29·2	−17·7	−29·6	+31·8	+18·9
896	B9	4·9	·0214	− 0·2	+ 1·9	+ 3·8	− 2·5	− 3·0	+ 8·1
1566	B9	5·2	·0186	+14·4	+16·9	−19·4	−11·8	+11·5	−18·0
1702	B8	3·2	·0204	− 2·5	+14·9	−20·5	−11·7	+20·4	− 3·3
2011	B8	4·5	·0112	− 5·6	+24·0	− 0·8	−24·1	+ 5·2	+ 6·6
2406	B8	4·0	·0141	−13·2	+26·4	−10·9	−25·9	+17·9	+ 8·1
3115	B9	4·4	·0269	+18·6	+ 9·0	+10·7	− 8·1	−17·5	− 6·6
4666	B8	4·8	·0098	− 2·0	−14·4	−61·4	−23·4	+53·6	−17·0
4934	B8*p*	4·6	·0107	− 4·8	+10·0	+ 1·4	−10·6	+ 1·9	+ 9·5
5199†	B9	4·4	·0269	− 3·2	+ 9·1	−20·4	− 6·1	+20·1	− 1·7
5349†	B8	5·8	·0062	−16·7	+16·2	−14·3	−15·8	+21·7	+11·6

TABLE VII.

P.G.C.	Type.	m.	ϖ (assumed).	u.	v.	w.	P.	Q.	N.
			"						
111	A5	6·1	·0347	− 1·7	+ 8·7	− 3·4	− 8·2	+ 4·5	+ 4·9
203	A2	3·9	·0501	+ 0·7	+14·8	+ 6·3	−15·4	− 4·6	+ 6·0
253	A3	5·2	·0316	− 6·3	+ 2·5	+ 4·5	− 4·0	− 0·7	+13·5
264	A5	4·5	·0725	+ 4·0	+16·8	+ 9·9	−17·5	− 9·3	+ 4·4
314	A5	2·8	·1585	+ 1·9	+10·3	+ 6·2	−10 9	− 5·5	+ 5·7
482	A5	3·1	·1380	− 4·0	+ 3·7	− 3·0	− 3·6	+ 4·8	− 7·9
550	A5	4·6	·0692	− 0·3	− 0·5	+ 1·3	+ 0·3	− 1·0	+ 7·5
560	A	4·3	·0302	+ 0·6	+ 7·9	+ 0·1	− 7·6	+ 0·2	+ 4·6
680–1	A2	4·4	·0398	+17·3	+ 5·7	− 9·9	− 1·9	+ 1·0	−14·2
690	A5	5·1	·0550	+ 9·7	+ 9·8	−23·3	− 4·8	+16·6	−14·4
730	A	5·0	·0219	+17·2	+11·0	− 9·9	− 7·1	+ 1·5	−15·0
806	A	4·8	·0240	+ 3·1	+ 6·7	+ 1·3	− 6·4	− 2·1	+ 3·2
986	A2	4·6	·0363	+ 2·7	+20·6	+12·4	−21·8	−10·6	+ 5·9
1012	A	5·3	·0191	−14·0	− 4·1	+ 2·4	+ 1·8	+ 4·2	+20·6
1130	A5	5·4	·0479	− 2·6	− 5·8	+18·3	+ 2·5	−15·3	+18·2
1363	A	6·8	·0096	+ 0·9	+18·0	+ 0·3	−17·5	+ 0·7	+ 2·5
1482	A*p*	2·7	·0525	− 4·0	+27·0	+10·1	−28·6	− 4·9	+ 9·6
1575	A	4·4	·0288	+ 1·1	− 4·8	− 1·3	+ 5·0	+ 0·2	+ 5·8
1690	A	1·9	·0912	− 0·8	−11·0	− 5·7	+11·6	+ 4·5	+ 6·6
1696	A	4·4	·0288	+ 0·5	+13·1	−19·5	− 9·7	+17·9	− 5·3
1952	A5	4·2	·0832	− 4·5	−12·0	+ 6·2	+10·2	− 4·2	+15·4
1968	A3*p*	4·8	·0380	−13·5	+30·7	−13·9	−29·6	+21·0	+ 6·3
2078	A2	5·0	·0302	+12·2	−13·8	−15·6	+17·4	+ 6·8	− 8·6
2426	A	5·4	·0182	+ 9·1	− 9·4	− 7·5	+11·6	+ 1·6	− 3·1
2637	A2	4·5	·0380	+ 7·0	− 4·7	− 9·9	+ 7·0	+ 5·0	− 3·3

TABLE VII.—*continued.*

P.G.C.	Type.	*m.*	ϖ (assumed).	*u.*	*v.*	*w.*	P.	Q.	N.
			"						
2692	A5	4·5	·0725	+ 3·0	− 5·7	− 4·4	+ 6·7	+ 2·0	+ 2·9
2754	A	4·9	·0229	− 1·6	+ 3·3	− 5·3	− 2·6	+ 5·7	+ 4·9
2928	A5	5·1	·0550	+17·2	− 0·2	− 4·5	+ 3·1	− 4·2	−10·5
2929	A2	4·6	·0363	+ 6·3	− 5·0	+ 5·4	+ 4·9	− 8·1	+ 4·4
2932	A	4·4	·0288	+12·1	− 0·8	− 0·3	+ 2·3	− 5·5	− 4·0
3092	A5	3·8	·1000	− 6·9	+ 5·2	−13·2	− 3·9	+15·3	+ 5·5
3283	A	4·8	·0240	+17·8	+16·6	− 7·0	−12·9	− 1·0	−15·3
3363†	A*p*	1·7	·0832	+ 6·8	− 5·2	− 9·5	+ 7·5	+ 4·7	− 2·7
3371†	A*p*	2·9	·0479	− 0·2	+23·5	+ 1·4	−23·2	+ 0·8	+ 3·0
3447	A5	4·7	·0661	− 8·7	+ 6·6	+ 5·8	− 8·5	− 0·5	+15·5
3474	A*p*	2·4	·0603	+ 7·4	− 7·8	−10·4	+10·1	+ 5·0	− 3·2
3475	A2	4·0	·0479	+ 9·0	−10·6	−13·1	+13·5	+ 6·4	− 5·3
3480	A5	4·0	·0912	+ 8·1	− 3·7	−11·4	+ 6·4	+ 5·9	− 5·1
3512†	A3	4·6	·0417	+ 8·1	+17·7	−14·1	−14·1	+10·0	−10·3
3612	A2	4·3	·0117	+10·6	+ 3·4	− 3·2	− 1·5	− 1·9	− 4·8
3626†	A	3·6	·0417	+ 4·9	+ 9·7	−15·3	− 6·5	+11·9	− 6·5
3654	A5	4·6	·0692	+12·0	+ 2·4	−16·4	+ 1·7	+ 8·9	−11·8
3667†	A5	4·8	·0631	+ 8·1	+18·4	−10·9	−15·3	+ 7·3	− 8·9
3672	A2	4·6	·0363	+ 1·9	+ 4·7	+ 4·0	− 5·0	− 4·0	+ 5·8
3752	A2	4·4	·0398	+ 9·2	− 1·0	− 4·9	+ 3·0	− 0·2	− 3·6
3814†	A2	4·6	·0363	−21·7	− 2·4	−22·7	+ 3·1	+30·0	+15·5
3825†	A	4·8	·0240	+23·7	+41·5	+ 4·3	−38·4	−11·7	−19·9
3855	A5	5·0	·0575	+11·0	−10·8	−16·6	+14·6	+ 8·5	− 8·6
3866	A*p*	4·7	·0209	+ 8·1	+20·8	− 4·5	−18·7	+ 1·7	− 6·5
3928	A2	3·1	·0725	+ 1·6	+ 3·2	− 8·3	− 1·6	+ 6·8	+ 0·7
4026	A	3·8	·0380	+19·3	+ 1·2	+ 6·2	+ 0·3	−14·4	− 7·8
4084	A3*p*	4·8	·1445	+ 6·1	+10·4	+12·8	−11·4	−13·3	+ 5·1
4269	A*p*	5·3	·0158	− 1·2	+18·1	−15·1	−15·5	+15·2	− 2·7
4360	A	2·6	·0661	+ 2·6	− 0·4	+ 6·5	− 0·3	− 6·9	+ 7·3
4391	A	4·8	·0240	− 5·8	+11·7	+ 5·3	−13·0	− 1·0	+11·8
4458	A5	5·0	·0575	+13·9	+10·4	−10·4	− 6·8	+ 3·3	−12·3
4462	A5	3·6	·1096	+ 3·4	+43·0	+ 8·6	−43·0	− 5·7	− 0·6
4475	A2	4·4	·0661	− 4·7	+35·3	+ 1·0	−35·4	+ 4·1	+ 4·6
4556	A	5·1	·0209	+ 3·5	+28·3	− 4·6	−26·6	+ 4·5	− 3·9
4581	A2	3·7	·0550	− 5·6	+25·8	+ 2·8	−26·4	+ 2·1	+ 7·9
4804	A	5·1	·0209	− 3·5	− 1·8	− 3·0	+ 1·9	+ 4·2	+ 8·5
4832	A2	2·7	·0871	+ 3·3	−17·9	−10·5	+19·5	+ 6·2	+ 2·1
4948	A2	4·6	·0363	− 3·4	+16·5	−23·7	−12·9	+23·7	− 4·4
4962	A8	4·9	·0661	− 1·6	+ 2·3	+ 0·3	− 2·5	+ 0·7	+ 7·7
5034	A3	5·5	·0275	+ 4·5	− 7·7	−17·1	+10·8	+12·3	− 3·8
5315	A5	3·6	·1097	+ 0·7	− 4·1	− 7·1	+ 5·2	+ 5·6	+ 3·4
5320†	A2	1·3	·1660	− 2·2	+ 2·7	− 3·6	− 2·3	+ 4·4	+ 6·3
5608	A2*p*	4·5	·0200	−10·5	+ 6·6	−22·1	− 4·4	+24·9	+ 4·5
5695	A5	5·5	·0457	−23·1	+ 8·5	+19·4	−14·2	− 5·4	+33·8
5831	A2	5·8	·0209	+ 0·3	+11·2	−19·7	− 7·9	+18·1	− 4·8
6062	A2*p*	4·8	·0174	+17·3	+ 4·6	−18·9	+ 0·6	+ 8·8	−18·1

5. Although the parallaxes assigned are purely empirical and obviously have no claim to individual accuracy, it is clearly better to derive them in accordance with a definite rule. Hence a formula of the type

$$\log \varpi = c - 0{\cdot}2m$$

has been adopted in dealing with the other classes of stars on similar lines. The parallaxes appearing in Table VI., which applies to the types B8 and B9, are based on the constants

$$c = 8{\cdot}95 \text{ for B8, } c = 9{\cdot}31 \text{ for B9.}$$

Similarly the parallaxes in Table VII., which refers to the outstanding type A stars, are based on the following constants:—

Type.	A.	A*p*.	A2.	A2*p*.	A3.	A3*p*.	A5.	A8.
c	9·34	9·26	9·48	9·20	9·54	10·12	9·76	9·80

The numbers of certain stars, as in Table V., are marked thus † in these tables. They correspond to the stars already marked in Tables III. and IV., and are included because it seemed desirable to see the effect of assigning parallaxes more in accordance with the general average.

6. Tables V.–VII. give, in addition to the type, visual magnitude, and parallax assumed, the three components of velocity (u, v, w) relative to the Sun and referred to equatorial axes, and also the three components (P, Q, N) the nature of which has been explained in § 3. In the case of these stars N is no longer zero, nor is the average value so small as was previously found for a particular group of stars in No. 4, § 11. The result of taking average values of N, with and without regard to sign, for the three classes of stars, may be shown thus:

	B–B5.	B8–B9.	A.	All.
With sign	+2·8	+2·4	+0·9	+1·7
Without sign	5·3	9·0	7·9	7·2

These are considerable, and it is impossible to decide how far the inaccuracy of the assigned parallaxes may be responsible, to what extent these components may be real, or what inference should be drawn as to the average magnitude of the ignored component in the case of that greater number of stars which are plausibly amenable to the law of galactic motion. The distribution between positive and negative components is not symmetrical, the excess of positive components being marked. These have the same direction as the corresponding component of the solar motion. This point is illustrated by combining the material from the three Tables V.–VII. and noting the number of cases, thus:

N	0	1	2	3	4	5	6	7	8	9	10	>10·5
+		2	5	7	8	9	12	6	10	3	3	13
−	2	1	4	7	8	7	1	3	1	3	3	11

The want of symmetry here appears to be local rather than general, occurring conspicuously only between +6 and +8

km./sec. This clearly suggests an excess of stars which share the motion of the Sun normal to the galaxy. The absence of small components is in one sense satisfactory. The general method followed may have been too comprehensive, but there is nothing to suggest that much suitable material has evaded its grasp. It will be noticed that in only two instances does the value of N exceed 20, while in half of the outstanding cases—in all 129 out of a total of 459—the value does not exceed 6 km./sec.

7. With the new material contained in Table I. two additional stars, Nos. 3512 and 3667, present themselves which may be attributed to the high velocity stream discovered by Mr. B. Boss, thus preserving the average of one in forty which has been found to characterise these stars. These are incorporated in the following Table VIII., in which all the results of this kind are collected, and the reduction is based on the assumption that the relative motion between each star and the Sun is 95 km./sec. The headings have all been sufficiently described.

TABLE VIII.

P.G.C.	Star.	Type.	m.	m_0.	ϖ.	u.	v.	w.	P.	Q.	N.
					"						
199	γ Cassiop.	Bp	2·2	−1·9	·0015	−19·0	+92·2	− 9·8	−91·2	+24·9	+1·6
264	θ ,,	A5	4·5	4·8	·0116	−13·3	+93·9	+ 6·3	−94·7	+ 8·2	+3·6
314	δ ,,	A5	2·8	3·7	·0153	−16·4	+93·6	− 0·3	−93·8	+15·5	+3·4
2581	h Carinæ	B5	4·2	−1·0	·0009	− 2·1	+93·7	+15·4	−94·5	− 5·1	−2·0
2862	θ ,,	B	3·0	−1·6	·0012	− 5·0	+94·6	+ 5·7	−94·2	+ 4·9	−4·0
3371	α^2 Can. Ven.	Ap	2·9	3·3	·0118	− 7·9	+93·9	+11·6	−94·8	+ 0·9	+1·3
3512	24 ,,	A3	4·6	3·5	·0060	−15·1	+93·5	− 7·5	−92·3	+21·1	−1·0
3626	α Draconis	A	3·6	0·8	·0027	−18·6	+93·1	− 5·0	−92·8	+20·6	+3·2
3667	ι Bootis	A5	4·8	4·5	·0086	− 2·9	+93·9	+14·4	−94·6	− 3·8	−1·9
3825	δ Libræ	A	4·8	2·8	·0039	−22·8	+92·0	− 6·6	−92·1	+23·9	+6·4
5199	κ Cephei	B9	4·4	0·3	·0015	−13·6	+94·0	− 3·2	−93·3	+16·6	−0·5
6173	ζ Sculpt.	B5	5·0	0·6	·0013	− 8·3	+94·6	− 2·9	−93·3	+13·9	−5·1
					Means	−12·1	+93·6	+ 1·5	−93·5	+11·8	+0·4

8. Whether the interpretation placed upon the motion of these stars as members of a fairly homogeneous and remarkable stream is really justified it is impossible to say. A definite answer requires an exact knowledge of the distances of the stars, and this is not attainable. If real, the stream is practically parallel to the plane of the galaxy and corresponds to a point far out along the axis of Schwarzschild's ellipsoid. What chiefly causes us to hesitate in accepting this view is the smallness of the parallaxes and the corresponding brightness of the absolute magnitudes. The latter are systematically brighter, in the average 3·5 magnitudes, than the means of their respective classes. This suggests that the velocities have been made large by making the parallaxes unduly small. It will be particularly difficult to establish the homogeneity

of the stream even if something of the kind be admitted. The original results, based on the idea of pure galactic motion, are not altogether out of the question, and would point to varying velocities still conforming to the preferred line of motion, though their differences have doubtless been exaggerated by the too rigid law which they have been forced to obey. Under this interpretation the absolute magnitudes are of the same order and are liable to the same objection. On these grounds the same twelve stars have also been treated by the third method of assigning empirical parallaxes in better accordance with what we should expect. The results will be found in Tables V.–VII. Under this treatment the stars seem to lose any marked connection.

9. In the earlier discussion of the A type stars it was noticed that a considerable number of stars in all parts of the sky shared the same type of motion which characterises the Taurus stream discovered by Lewis Boss. With the increased material now at our disposal it is desirable to revise the list previously given, and the result is shown in Table IX.

TABLE IX.

P.G.C.	Star.	Type.	m.	m_0.	ϖ.	P.	Q.
					"		
629	μ Ceti	A5	4·4	7·4	·040	−41·9	+2·6
1022	64 Tauri	A2	4·8	7·6	·036	−39·7	−3·0
1026	κ^1 ,,	A3	4·4	6·8	·030	−43·0	−4·9
1029	68 ,,	A	4·2	7·4	·043	−36·4	−6·2
1194	ι ,,	A5	4·7	8·2	·049	−40·4	−8·4
2327	γ Cancri	A	4·7	6·2	·0203	−37·7	+3·8
2729	λ Ursæ Maj.	A	3·5	5·2	·022	−39·5	+0·2
3302	γ Centauri	A	2·4	4·4	·025	−38·0	−1·3
4162	τ Herculis	B5	3·9	2·2	·0045	−37 9	−6·2
4462	ξ Serpentis	A5	3·6	(8·8)	(·1096)	(−43·0)	(−5·7)
4566	π Pavonis	A5	4·4	6·9	·031	−35·5	−3·0
4802	θ^1 Serpentis	A5	4·5	6·4	·0240	−39·9	+0·9
5062	α Aquilæ	A5	0·9	6·2	·113	−42·6	−7·9
5086	13 Vulpec.	A	4·5	3·6	·0067	−40·0	−3·5
5171	θ Aquilæ	A	3·4	2·9	·008	−34·3	0·0
5188	29 Cygni	A	5·0	5·6	·0130	−40·7	−3·3
5332	ι Microscop.	A5	5·1	7·7	·0335	−33·2	−2·3
5467	θ Indi	A5	4·6	6·1	·020	−34 9	+0·8
5910	ρ Pegasi	A	5·0	4·8	·009	−40·2	−3·3
					Means	−38·9	−2·7
					(L. Boss)	−45·3	−0·8

The elements given by Boss (*A.J.*, 604), namely, 45·6 km. towards the point (91°·6, + 6°·9), give N = + 1·4, showing that the motion is very approximately of galactic type. The values of P are systematically in defect, even for the three stars 1022, 1026,

and 1029, which are accepted members of the stream. To some extent this is a mere matter of linear scale, since the computed radial velocities of the three stars average +39·8, whereas the observed values of V–K average +37·5. A corresponding change in the mean value of P for the above group would give −41·2 instead of −38·9. Only in one instance is an empirical parallax employed, and it may be excessive in amount. But the star, ξ Serpentis, is quite certainly moving with a velocity of galactic type for N = −0·6, and this value is independent of any assumption as to the parallax, the proper motion itself being parallel to the Milky Way. On the whole, the evidence of the three certain members of the Taurus stream is consistent with the belief that the values of P and Q represent at least a serious first approximation to the projection of the motions on the plane of the Milky Way.

10. A more exacting test of this idea is furnished by the Ursa Major stream as extended by Hertzsprung. The results for these stars are collected in Table X.

TABLE X.

P.G.C.	Star.	Type.	m.	m_0.	ϖ.	P.	Q.	N.	C.	V−K.
					"					
1220	β Eridani	A2	2·9	6·3	·0470	+13·3	+ 2·4	...	− 7·5	− 9
1478	β Aurigæ	A*p*	2·1	6·1	·0618	+15·7	+ 9·5	...	−16·0	−19·
1732	α Canis Maj.	A	−1·6	7·3	·596	+11·6	+ 2·4	...	− 8·5	− 8·
2930	β Ursæ Maj.	A	2·4	4·6	·0277	+18·7	+ 2·8	...	−16·1	−13
2972	δ Leonis	A2	2·6	3·6	·0157	+49·8	+42·2	...	−14·4	−19
3117	γ Ursæ Maj.	A	2·5	5·7	·0430	+12·3	+ 3·5	...	−15·0	−10
3190	δ ,,	A2	3·4	4·7	·0183	+30·2	+ 2·3	...	−14·4	−12
3363	ε ,,	A*p*	1·7	0·9	·007	+76·0	− 2·0	...	−13·2	−11
				(6·3)	(·0832)	(+ 7·5)	(+ 4·7)	(−2·7)		
3474	ζ^1 ,,	A*p*	2·4	(6·3)	(·0603)	(+10·1)	(+ 5·0)	(−3·2)	−12·2	−11
3475	ζ^2 ,,	A2	4·0	(7·4)	(·0479)	(+13·5)	(+ 6·4)	(−5·3)	−12·2	−13
3961	α Coronæ	A	2·3	6·3	·0628	+ 9·5	+ 3·0	...	− 2·2	+ 0
					(Hertzsprung)	+13·8	+ 5·5	−4·3		

The figures in the last line are based on Hertzsprung's elements for the stream (*Ap. J.*, xxx. p. 136), a relative velocity of 18·4 towards the convergent (307°·8, −40°·2). This is a severer and more thorough test than that provided by the Taurus stream, because the number of stars available for comparison is greater, because the component N is larger, and because the total relative velocity is less. In fact, the stars belong to Stream II., in which the motions are opposed to the solar motion (as will be seen more clearly immediately). The result is at least fairly satisfactory. One star, δ Leonis, is clearly astray, and δ Ursæ Majoris is not well represented. The motion of ε Ursæ Majoris originally found and

the absolute magnitude corresponding to the same parallax were naturally suspected, and the alternative parallax assigned gives a much better result. The empirical parallaxes assigned to the components of ζ Ursæ Majoris in accordance with rule also seem quite appropriate. On the whole we have again, as the outcome of the general method, at least a useful approximation to the motion of the system projected on the plane of the galaxy. At the time when Hertzsprung's paper was published only a certain proportion of the radial velocities were known. They are now given in the last column of Table X., and in the preceding column will be found his predicted values, taken from Table I. of his paper (*loc. cit.*, p. 137). It will be noticed that the agreement is generally good without being exact. There are three other stars in Hertzsprung's list, but these belong to type F.

11. The nature of the systematic motions projected on the plane of the galaxy, so far as they are indicated by the whole of the present work, will be best seen by plotting the values of P and Q as point co-ordinates in a series of diagrams. This is done for the three divisions of type B–B5, B8–B9, and A respectively in the three diagrams. Such diagrams may perhaps be called hodograms, since they attempt to represent the velocities of a number of stars at one time, whereas the hodograph represents the velocity of a single point at successive instants. In all these diagrams the origin corresponds to the Sun, to which the stellar velocities are relative. The axes of P and Q correspond to the line of preferential motion and the line at right angles. The figured divisions on these axes represent kilometres per second. The points marked with a simple dot are those which result directly from the hypothesis of galactic motion, and their co-ordinates are taken from Tables II.–IV. The points which are marked by a cross (+) are based on mean parallaxes, and their co-ordinates are taken from Tables V.–VII. The point surrounded by a circle on each diagram denotes the mean position of the dots, and its displacement from the origin represents the projection of the motion of the system relative to the Sun. It will be noticed that the change in the position of this point is progressive with advancing spectral type.

12. Making abstraction of the origin of reference, we have the relative motions of the stars among themselves in the several systems. The respective types of motion are very distinct, and on the whole in good accordance with what other work has led us to expect. Fig. 1 for the earlier B types shows a single cluster of points very few of which extend beyond the origin. The relative motions indicated are for the most part small and show no marked organic tendencies. In particular, there is no tendency towards the P-axis, which represents the two-stream line of preference. If any distinct line is favoured it is a line making an angle of about 140° with the positive direction of the P-axis, and this corresponds with the projection of Halm's stream O. Unfortunately very few stars are available for insertion in fig. 2, which in the

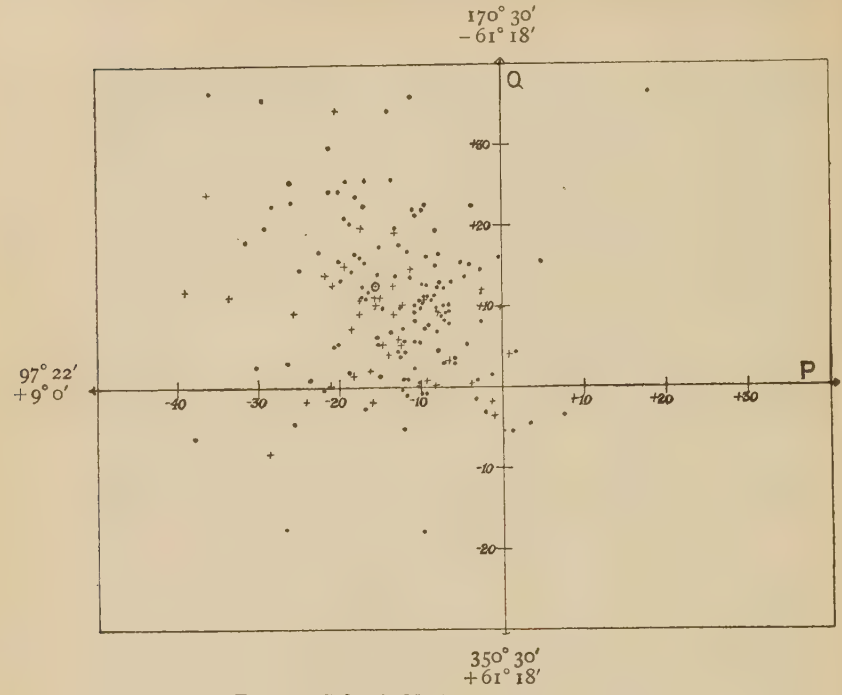


FIG. 1.—Galactic Motions of Stars of Type B–B5.

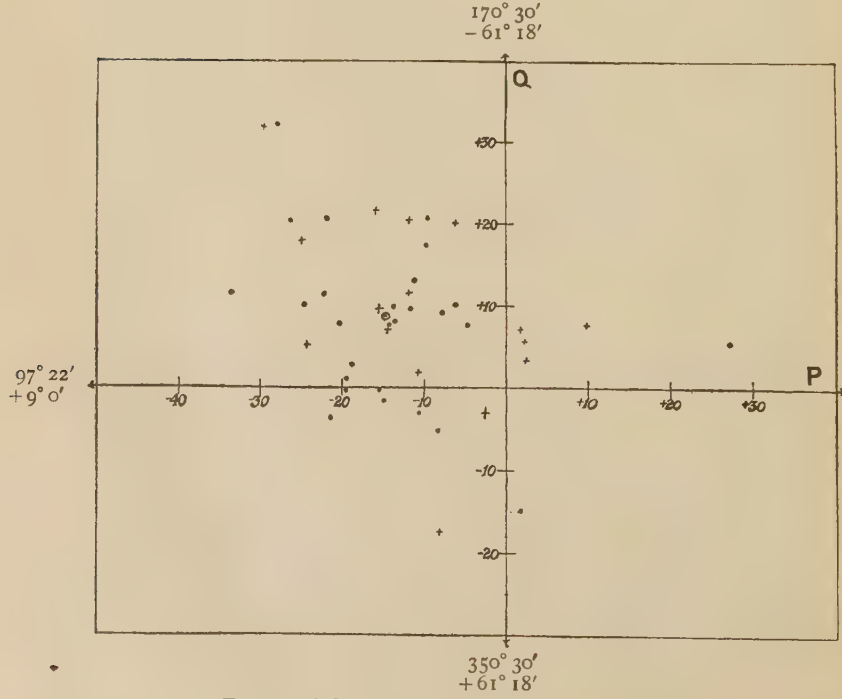


FIG. 2.—Galactic Motions of Stars of Type B8–B9.

main resembles fig. 1 but at the same time shows a much greater diffuseness and a slight tendency towards the features which become pronounced in fig. 3. Here the distribution seems to be resolvable into two main types, a survival of stars still possessing the characteristics of the earlier B class in the second quadrant, and a distinct arrangement along the P-axis. The two marked (×) points T and U respectively represent the projections of the motion of the Taurus stream and the Ursa Major stream according to L. Boss and Hertzsprung. The line may stretch out to the left

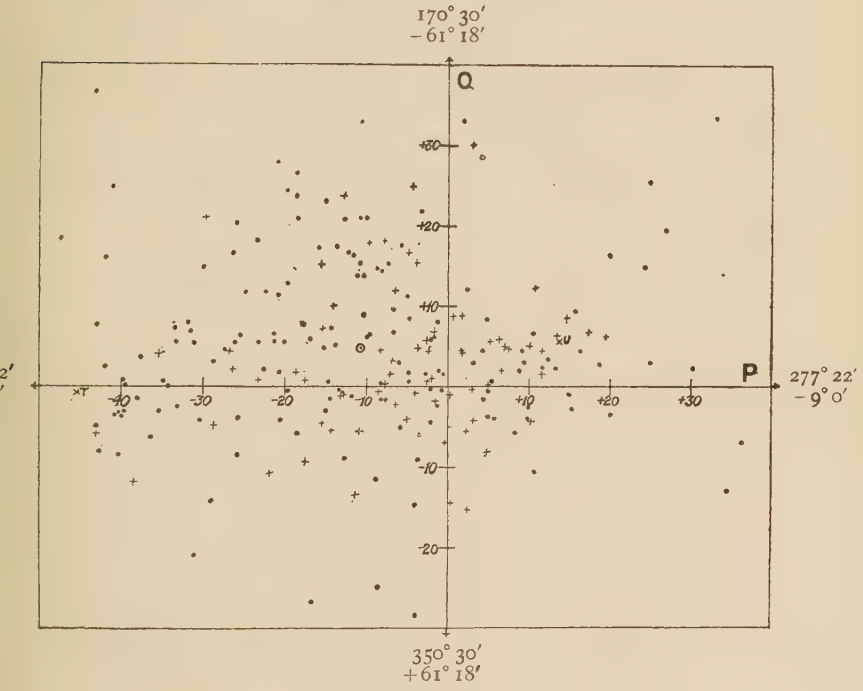


Fig. 3.—Galactic Motions of Stars of Type A.

beyond the limits of the diagram so as to embrace the stars attributed to the stream of B. Boss. But the majority of the points lie between the limits T and U. Between these two positions the distribution is fairly uniform. It would be possible to divide the line into two parts and to call the portion on the left Stream I. and that on the right Stream II. But there is no marked break of continuity, and the division would be arbitrary. With the material before us in this form it is difficult to retain the idea of distinct streams, as developed in the earlier papers of this series. Not that stars cannot be selected which appear to possess a common type of motion, but the boundaries of the type cannot be defined, and such a group possesses little interest unless its members share

some other characteristic such as actual physical contiguity. It may be that the distinction between groups is obscured by the operation of the too rigid hypothesis to which the motions have been forced to conform, and this applies in particular to the evidence bearing on the resolution of Stream I. and Stream II. But this defect is inherent in the method. The diagrams convey in the clearest possible form a summary of the results obtained. In fig. 3 it will be noticed that a number of points fall near the origin: it is very natural that the corresponding stars, some of which may well share the velocity of the Sun itself, should have yielded negative parallaxes in many cases.

13. A summary of this paper is as follows:—

§ 1. References to the earlier papers of the series, with errata.

§ 2. Explanation of Table I., which gives additional material on stars of type A.

§ 3. The scheme of transformation to a galactic system of co-ordinates. Explanation of Tables II.–IV., which give the galactic components of velocity for all stars conforming with the primitive hypothesis.

§ 4. Explanation of Table V., giving for stars of type B–B5 the result of assigning empirical parallaxes in all outstanding cases.

§ 5. Explanation of Tables VI. and VII., which apply similarly to the stars of types B8–B9 and A.

§ 6. Examination of the normal component N as given in Tables V.–VII.

§§ 7, 8. Discussion of twelve stars given in Table VIII. and tentatively attributed to the stream of high velocity discovered by B. Boss.

§ 9. Revised list of stars (Table IX.) whose type of motion may resemble that of the Taurus stream.

§ 10. Results (Table X.) for the stars which are members of Hertzsprung's extension of the Ursa Major stream.

§§ 11, 12. Description of the three diagrams representing the galactic components of the stellar motions as derived in the course of the present work.

Dunsink Observatory:
1913 *April* 7.

On a Family of Oscillating Orbits of Short Period. By Harley R. Willard, Ph.D., Assistant Professor of Mathematics in the University of Maine. (Plate 18.)

(*Communicated by Ernest W. Brown, F.R.S.*)

1. It is known that if three bodies, acted upon only by forces due to their mutual attractions under the Newtonian Law, are placed at the vertices of an equilateral triangle and given proper velocities, they will retain their original relative positions. Furthermore, it is known that if certain conditions with regard to their relative masses are satisfied their motion will be stable. * Sir George Darwin has discussed several classes of the motion of the third body in a periodic orbit when its mass is infinitesimal as compared with the masses of the other two bodies. † Professor Ernest W. Brown has treated the case of the motion in periodic orbits where the asteroid moves about one of the triangular points of equilibrium in an orbit of *long* period. ‡G. W. Hill was the first, however, to discuss periodic orbits, and since then Poincaré and others have treated different phases of such motion. It is the purpose of this paper to investigate the particular case in the Problem of Three Bodies where the asteroid oscillates in a periodic orbit of ***short*** period about one of the triangular points of equilibrium, and to compute the possible orbits in which it might move under the assumed conditions.

2. The conditions that I shall assume are the same as those imposed by Sir George Darwin and by Professor Brown. The origin of co-ordinates is the Sun, and the x-axis passes through Jupiter which revolves in a circle with the uniform angular velocity $n=(1+m)^{\frac{1}{2}}$. The distance from Jupiter to the Sun is unity, the mass of the Sun is unity, the mass of Jupiter is m, and the mass of the third body, the asteroid, is infinitesimal.

The method used by Professor Brown in the paper referred to for working out the orbit of long period about the triangular points of equilibrium was not available for this work, and the method adopted was that of Hill.§ This method was used by Professor Brown‖ in his second paper on Periodic Orbits. The method consists in the use of complex co-ordinates, in which the solution is supposed to be expanded in powers of an arbitrary parameter.

Let ρ be the distance of the asteroid from Jupiter, and r its distance from the Sun.

$$r^2=x^2+y^2 \quad \text{and} \quad \rho^2=(x-1)^2+y^2,$$

where the distance between Jupiter and the Sun is taken to be unity.

* Sir George Darwin: *Acta Mathematica*, vol. xxi. p. 103.

† Professor Ernest W. Brown: "On a Family of Periodic Orbits in the Problem of Three Bodies," *Monthly Notices of R.A.S.*, March 1911.

‡ G. W. Hill: *Collected Works.*

§ G. W. Hill: *Collected Works*, vol. i. p. 284 *et seq.*

‖ Professor Ernest W. Brown: "On the Oscillating Orbits about the Triangular Equilibrium Points in the Problem of Three Bodies," *Monthly Notices of R.A.S.*, April 1911.

Under these conditions the equations of motion referred to rotating axes are

$$\left.\begin{aligned} \ddot{x} - 2n\dot{y} &= \frac{\partial \Omega}{\partial x} \\ \ddot{y} + 2n\dot{x} &= \frac{\partial \Omega}{\partial y} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where

$$2\Omega = \frac{2}{r} + r^2 + m\left(\frac{2}{\rho} + \rho^2\right).$$

I shall adopt Hill's transformation by setting

$$u = x + iy, \quad s = x - iy, \quad \frac{d}{dt} = i\mathrm{D},$$

where $$i = \sqrt{-1}.$$

Then $$r^2 = us \quad \text{and} \quad \rho^2 = us - u - s + 1.$$

Making this transformation, the equations of motion become

$$\left.\begin{aligned} \mathrm{D}^2u + 2n\mathrm{D}u + 2\frac{\partial \Omega}{\partial s} &= 0 \\ \mathrm{D}^2s - 2n\mathrm{D}s + 2\frac{\partial \Omega}{\partial u} &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

If we turn the axes through a constant angle $\frac{1}{3}\pi - \beta$ and let x, y, u, s be the co-ordinates referred to the new set of axes, the equations of motion, (2) will not be changed.

Let $$u_0 = e^{\frac{1}{3}\pi i} \quad \text{and} \quad s_0 = e^{-\frac{1}{3}\pi i}$$

be the co-ordinates of the upper equilibrium point referred to the old set of axes. Then $e^{\beta i}$ and $e^{-\beta i}$ will be the co-ordinates of the same point referred to the new axes.

Put $$u = e^{\beta i} + u_1 \quad \text{and} \quad s = e^{-\beta i} + s_1$$

and expand equations (2) in powers of u_1 and s_1. This process gives

$$\left.\begin{aligned} &(\mathrm{D}^2 + 2n\mathrm{D} + a_1)u_1 + b'_1s_1 + \tfrac{1}{2}b_2u_1^2 + b'_2u_1s_1 + \tfrac{1}{2}c'_2s_1^2 + \tfrac{1}{3}a_3u_1^3 \\ &\quad + b_3u_1^2s_1 + a'_3u_1s_1^2 + \tfrac{1}{3}c'_3s_1^3 + \ \cdot\ \cdot\ \cdot\ = 0 \\ &(\mathrm{D}^2 - 2n\mathrm{D} + a'_1)s_1 + b_1u_1 + \tfrac{1}{2}c_2u_1^2 + b_2u_1s_1 + \tfrac{1}{2}b'_2s_1^2 + \tfrac{1}{3}c'_3u_1^3 \\ &\quad + a_3u_1^2s_1 + b_3u_1s_1^2 + \tfrac{1}{3}a'_3s_1^3 + \ \cdot\ \cdot\ \cdot\ = 0 \end{aligned}\right\} \quad . \quad (3)$$

We are to solve equations (3) by continued approximation. Here a_1, b_1, a_2, b_2, etc., are certain derivatives of 2Ω at the equilibrium point, and we shall give their values later.

3. If we neglect all powers of u_1 and s_1 but the first, the equations become linear; and if we put $u_1 = \mathrm{A}e^{\lambda it}$, $s_1 = \mathrm{B}e^{\lambda it}$ and divide out the factor $e^{\lambda it}$, we get

$$\mathrm{A}\lambda^2 + 2n\mathrm{A}\lambda + \mathrm{A}a_1 + b'_1\mathrm{B} = 0$$
$$\mathrm{B}\lambda^2 - 2n\mathrm{B}\lambda + \mathrm{B}a'_1 + b_1\mathrm{A} = 0.$$

Eliminate $\frac{A}{B}$ and we obtain the equation

$$\lambda^4 - (1+m)\lambda^2 + \frac{27}{4}m = 0 \quad . \quad . \quad . \quad (4)$$

from which the values of λ may be determined. The roots are real and all the solutions are periodic provided $(1+m)^2 > 27\,m$. In our problem $m = \frac{1}{1048}$, and this condition is satisfied. If we solve this equation for λ we shall see that one value of λ is nearly equal to n, while the other is of the order $\sqrt{m}$. The latter case is that treated by Professor Brown, and I shall make no further mention of it. The former is the case considered here.

4. In this work I have made use as far as possible of the notation of Professor Brown in his second paper. Most of the coefficients a_1, b_1, . . . are complex quantities. The notation is such that a coefficient with an accent is conjugate to the same letter without the accent. Really, it was not necessary to use both of equations (3); for, if we change the sign of i, the accented letters become unaccented, and *vice versa*, and u and s are interchanged. The use of both, however, gives a valuable check on the accuracy of the work. We have

$$a_1 = 2\frac{\partial^2 \Omega}{\partial u \partial s}\,;\quad b'_1 = 2\frac{\partial^2 \Omega}{\partial s^2}\,;\quad b_1 = 2\frac{\partial^2 \Omega}{\partial u^2}\,.$$

Hence a_1 and a'_1 are real and equal, and b_1 and b'_1 are complex. We shall determine β so that b_1 and b'_1 are also real and equal. It will be noticed that

$$\frac{\partial^{m+n}\Omega}{\partial u^m \partial s^n}$$

is real when

$$|m-n| = 0 \text{ or } 2.$$

Hence a_1, b_1, a_2, b_2 are real and b_2, c_2, c_3 are complex. We must now determine β under the conditions just imposed. Professor Brown in his second paper has made this determination, and we have

$$e^{2\beta i} = \frac{1 - m\mu_0}{\sqrt{1 - m + m^2}}$$

from which

$$\tan 2\beta = \frac{m\sqrt{3}}{2-m}\,.$$

If we carry out the computation, putting $m = \frac{1}{1048}$, we find that

$$\beta = 1'\ 25'',\ \text{nearly.}$$

That is, the new x-axis passes nearly through the equilibrium point and the Sun. The other derivatives, and thence the other coefficients, in (3) can now be computed. When this is done, and

we expand in powers of m and neglect terms containing powers of m higher than the second, we have

$$a_1 = a'_1 = 2\frac{\partial^2 \Omega}{\partial u \partial s} = \tfrac{3}{2}(1+m)$$

$$b_1 = 2\frac{\partial^2 \Omega}{\partial u^2} = \tfrac{3}{16}(8 - 4m + 3m^2)$$

$$b'_1 = 2\frac{\partial^2 \Omega}{\partial s^2} = \tfrac{3}{16}(8 - 4m + 3m^2)$$

$$b_2 = 2\frac{\partial^3 \Omega}{\partial u^2 \partial s} = -\tfrac{3}{4}(1 + ms_0)e^{-\beta i}$$

$$= -\tfrac{3}{4}(1 + \tfrac{1}{4}m + \tfrac{1}{2}ms_0 + \tfrac{17}{32}m^2 - \tfrac{1}{2}m^2 s_0)$$

$$b'_2 = 2\frac{\partial^3 \Omega}{\partial u \partial s^2} = -\tfrac{3}{4}(1 + mu_0)e^{\beta i}$$

$$= -\tfrac{3}{4}(1 + \tfrac{1}{4}m + \tfrac{1}{2}mu_0 + \tfrac{17}{32}m^2 - \tfrac{1}{2}m^2 u_0)$$

$$c_2 = 2\frac{\partial^3 \Omega}{\partial u^3} = -\tfrac{15}{4}(1 - m)e^{-3\beta i}$$

$$= -\tfrac{15}{4}(1 - \tfrac{1}{4}m - \tfrac{3}{2}ms_0 - \tfrac{39}{32}m^2 + \tfrac{3}{4}m^2 s_0)$$

$$c'_2 = 2\frac{\partial^3 \Omega}{\partial s^3} = -\tfrac{15}{4}(1 - m)e^{3\beta i}$$

$$= -\tfrac{15}{4}(1 - \tfrac{1}{4}m - \tfrac{3}{2}mu_0 - \tfrac{39}{32}m^2 + \tfrac{3}{4}m^2 u_0)$$

$$a_3 = \frac{\partial^4 \Omega}{\partial u^3 \partial s} = \tfrac{15}{16}(1 - \tfrac{1}{2}m + \tfrac{3}{8}m^2)$$

$$a'_3 = \frac{\partial^4 \Omega}{\partial u \partial s^3} = \tfrac{15}{16}(1 - \tfrac{1}{2}m + \tfrac{3}{8}m^2)$$

$$b_3 = b'_3 = \frac{\partial^4 \Omega}{\partial u^2 \partial s^2} = \tfrac{9}{16}(1 + m)$$

$$c_3 = \frac{\partial^4 \Omega}{\partial u^4} = \tfrac{105}{16}(1 - ms_0)e^{-4\beta i}$$

$$= \tfrac{105}{16}(1 + m - 3ms_0 - 3m^2)$$

$$c'_3 = \frac{\partial^4 \Omega}{\partial s^4} = \tfrac{105}{16}(1 - mu_0)e^{4\beta i}$$

$$= \tfrac{105}{16}(1 + m - 3mu_0 - 3m^2).$$

5. According to the well-known method, we proceed to solve (3) by assuming that a solution can be obtained in terms of the sines and co-sines of an angle λt and its multiples and of the powers of an arbitrary parameter. The subscript will denote the power of the parameter present in any given coefficient. Assume, then, that

$$u_1 = (P_1 + P_3)e^{\lambda t i} + (Q_1 + Q_3)e^{-\lambda t i} + P_2 e^{2\lambda t i} + Q_2 e^{-2\lambda t i} + R_2 + N_3 e^{3\lambda t i} + M_3 e^{-3\lambda t i} + \ldots \quad (5)$$

$$s_1 = (P'_1 + P'_3)e^{-\lambda t i} + (Q'_1 + Q'_3)e^{\lambda t i} + P'_2 e^{-2\lambda t i} + Q'_2 e^{2\lambda t i} + R'_2 + N'_3 e^{-3\lambda t i} + M'_3 e^{3\lambda t i} + \ldots \quad (5a)$$

For the present we shall not use the terms in $e^{\pm 3\lambda ti}$, since the coefficients that we are to determine first do not depend on the coefficients of these terms.

With this limitation, then, substitute (5) and (5*a*) in (3) and equate to zero the coefficients of $e^{\lambda ti}$. Then

$$\left.\begin{aligned}(P_1+P_3)(\lambda^2+2n\lambda+a_1)+b_1(Q'_1+Q'_3)+S_3=0\\(Q'_1+Q'_3)(\lambda^2-2n\lambda+a_1)+b_1(P_1+P_3)+T_3=0\end{aligned}\right\} \quad . \quad (6)$$

where S_3 and T_3 represent terms of at least the third order.

If we retain only terms to the third order inclusive, since

$$n^2=1+m,\ b_1{}^2=\tfrac{9}{4}(1-m+m^2),\ a_1=\tfrac{3}{2}(1+m),$$

we have *

$$\left(\lambda^2-\frac{n^2}{2}\right)^2-\tfrac{1}{4}(1+m)^2+\tfrac{27}{4}m=b_1\frac{S_3P_1+T_3Q'_1}{P_1Q'_1} \quad . \quad . \quad (7)$$

It is the right-hand member of (7) that we have to compute in order to obtain the correction for λ due to the retention of the higher order terms.

For the first approximation, if in (7) we place the right-hand member equal to zero, we have

$$\lambda=1-\tfrac{23}{8}m \quad . \quad . \quad . \quad . \quad (8)$$

$$n=1+\frac{m}{2} \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

neglecting m^2 and higher powers. If we neglect these terms of third order we see that P_1 and Q_1 are real, and therefore

$$P_1=P'_1, \qquad Q_1=Q'_1$$

and

$$Q_1=-3(1-\tfrac{3}{2}m)P_1=-2{\cdot}9957P_1 \quad . \quad . \quad (8b)$$

6. To obtain the second approximation equate to zero the coefficients of e^{0ti} and $e^{\pm 2\lambda ti}$ after substituting (5) and (5*a*) in (3). This gives

$$\left.\begin{aligned}a_1R_2+b_1R'_2+S_2=0 \quad . \quad . \quad (9)\\(4\lambda^2+4n\lambda+a_1)P_2+b_1Q'_2+T_2=0 \quad . \quad . \quad (10)\\(4\lambda^2-4n\lambda+a_1)Q'_2+b_1P_2+U_2=0 \quad . \quad . \quad (11)\end{aligned}\right\}$$

where

$$\left.\begin{aligned}S_2=(b_2+c'_2)P_1Q_1+b'_2(P_1{}^2+Q_1{}^2) \quad . \quad . \quad (9')\\2T_2=b_2P_1{}^2+2b'_2P_1Q_1+c'_2Q_1{}^2 \quad . \quad . \quad (10')\\2U_2=c_2P_1{}^2+2b_2P_1Q_1+b'Q_1{}^2 \quad . \quad . \quad (11')\end{aligned}\right\}$$

There are corresponding sets of equations referred to by the letter *a* which are obtained by interchanging accented and unaccented letters.

* Prof. Brown, *loc. cit.*, Second Paper.

S_3 and T_3 used in (6) have the values

$$\begin{aligned} S_3 = {} & b_2(R_2P_1 + P_2Q_1) + b'_2(P_1R'_2 + Q_1Q'_2 + P_1P_2 + Q_1R_2) \\ & + c'_2(P_1Q'_2 + Q_1R'_2) + a_3P_1^2Q_1 + b_3(P_1^3 + 2P_1Q_1^2) \\ & + a'_3(Q_1^3 + 2P_1^2Q_1) + c'_3P_1Q_1^2 \quad . \quad . \quad . \quad . \quad (12) \end{aligned}$$

$$\begin{aligned} T_3 = {} & c_2(P_1R_2 + P_2Q_1) + b_2(P_1R'_2 + Q_1Q'_2 + P_1P_2 + Q_1R_2) \\ & + b'_2(P_1Q'_2 + Q_1R'_2) + c_3P_1^2Q_1 + a_3(P_1^3 + 2P_1Q_1^2) \\ & + b_3(Q_1^3 + 2P_1^2Q_1) + a'_3P_1Q_1^2 \quad . \quad . \quad . \quad . \quad (13) \end{aligned}$$

From (12) and (13) by making use of (9′), (10a′), (9a′), (11a′), we obtain

$$\begin{aligned} & P_1S_3 + Q_1T_3 = R_2S'_2 + R'_2S_2 + 2P_2T'_2 + 2Q'_2U'_2 \\ & + P_1^2Q_1^2(c_3 + c'_3 + 2b_3) + (P_1^2 + Q_1^2)[4a_3P_1Q_1 + b_3(P_1^2 + Q_1^2)] \quad (14) \end{aligned}$$

If we eliminate R_2, R'_2, P_2, Q'_2, from this expression by means of equations (9), (9a), (10), (11), we shall have it in the form

$$\begin{aligned} P_1S_3 + Q_1T_3 = {} & -\tfrac{1}{2}\frac{(S_2 + S'_2)^2}{a_1 + b_1} + \tfrac{1}{2}\frac{(S_2 - S'_2)^2}{a_1 - b_1} \\ & + \frac{2}{F}[b_1(U_2T'_2 + U'_2T_2) - T_2T'_2(4\lambda^2 - 4n\lambda + a_1) \\ & - U_2U'_2(4\lambda^2 + 4n\lambda + a_1)] + 4a_3P_1Q_1(P_1^2 + Q_1^2) \\ & + b_3(P_1^2 + Q_1^2)^2 + (c_3 + c'_3 + 2b_3)P_1^2Q_1^2 \quad . \quad (15) \end{aligned}$$

where

$$\begin{aligned} F &= (4\lambda^2 + a_1)^2 - 16n^2\lambda^2 - b_1^2 \\ &= 12 - \tfrac{6\,3\,3}{4}m \quad . \quad . \quad . \quad . \quad . \quad (16) \end{aligned}$$

To compute S_2, T_2, etc., we make use of the values given by the first approximation. The computation is long, and I shall not give the details here. When this computation is completed, we have, after neglecting the second and higher powers of m,

$$\begin{aligned} P_1S_3 + Q_1T_3 = {} & [-\tfrac{1}{3}(8 - 15m) - \tfrac{1}{2}(\tfrac{8\,4\,1}{1\,2}m) \\ & + \tfrac{1}{6}(-32 + \tfrac{1\,5\,3}{2}m) + \tfrac{1}{1\,6}(128 + 113m)]P_1^2Q_1^2 \\ = {} & -\tfrac{4\,9\,1}{4\,8}mP_1^2Q_1^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (17) \end{aligned}$$

Since $P_1^2 + Q_1^2 = -3{\cdot}3295P_1Q_1$,

$$b_1 = \tfrac{3}{1\,6}(8 - 4m + 3m^2),$$

we have, after substituting (17) in (7),

$$\left(\lambda^2 - \frac{n^2}{2}\right)^2 = \tfrac{1}{4}n^4\left[1 - \frac{27m}{(1+m)^2} + \tfrac{1}{5\,7}(P_1^2 + Q_1^2)\right] \quad . \quad (18)$$

nearly.

Equation (18) gives us an expression for calculating the correction to be applied to λ which is due to terms of the third order. It is now possible to compute the values of the coefficients in (5) and (5a).

7. Making use of (8*b*) we easily derive

$$P_1^2 + Q_1^2 = (-\tfrac{10}{3} + 4m)P_1Q_1 \quad . \quad . \quad . \quad (a)$$
$$P_1^2 - Q_1^2 = (\tfrac{8}{3} - 5m)P_1Q_1 \quad . \quad . \quad . \quad (b)$$

from which

$$P_1^2 = \left(-\tfrac{1}{3} - \frac{m}{2}\right)P_1Q_1 \quad . \quad . \quad . \quad (c)$$
$$Q_1^2 = (-3 + \tfrac{9}{2}m)P_1Q_1 \quad . \quad . \quad . \quad (d)$$

If we use (*a*), (*c*), (*d*), together with values already obtained for b_2, b'_2, c_2, etc., in equations (9′), (10′), (11′) and (9*a*′), (10*a*′), (11*a*′), we deduce

$$S_2 = -\tfrac{3}{4}[\tfrac{8}{3} + \tfrac{8}{3}m - \tfrac{29}{3}mu_0 + \tfrac{601}{48}m^2 + \tfrac{95}{12}m^2u_0]P_1Q_1 \quad . \quad (19)$$
$$T_2 = [5 - \tfrac{39}{4}m - \tfrac{71}{8}mu_0]P_1Q_1 \quad . \quad . \quad . \quad . \quad . \quad (20)$$
$$U_2 = [1 - \tfrac{17}{8}m + \tfrac{15}{8}mu_0]P_1Q_1 \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The values of the corresponding accented functions (19*a*), (20*a*), (21*a*) are obtained by putting s_0 for u_0 in (19), (20), (21) respectively.

We now combine (19), (19*a*) with (9) and (9*a*), to obtain the expressions for R_2 and R'_2. It is necessary to retain terms in m^2 in the expressions for S_2, S'_2 because in solving (9), (9*a*), a factor m appears in the denominator. This gives

$$R_2 = \tfrac{1}{18}[41 - 58u_0 - \tfrac{117}{4}m + 33mu_0]P_1Q_1 \quad . \quad . \quad (22)$$

If we now substitute (20), (21) in (10), (11) and (20*a*), (21*a*) in (10*a*), (11*a*) with the values previously found for a_1, b_1, n, λ, we have, after reducing,

$$P_2 = \tfrac{1}{2}[-1 - \tfrac{45}{32}m + \tfrac{43}{16}mu_0]P_1Q_1 \quad . \quad . \quad . \quad (23)$$
$$Q_2 = \tfrac{1}{12}[-2 - \tfrac{395}{16}m + \tfrac{249}{8}mu_0]P_1Q_1 \quad . \quad . \quad (24)$$

To obtain (22*a*), (23*a*), (24*a*) substitute s_0 for u_0 in (22), (23), (24).

By substituting (22), (22*a*), (23), (23*a*), (24), (24*a*) and the values for b_2, b'_2 . . . in (12) and (13) and reducing, we have

$$S_3 = \tfrac{1}{8}[-152 + 232u_0 + \tfrac{3249}{8}m - \tfrac{1465}{4}mu_0]P_1^2Q_1 \quad . \quad (12')$$
$$T_3 = [\tfrac{10}{3} - \tfrac{29}{3}s_0 - \tfrac{6851}{192}m + \tfrac{73}{96}ms_0]P_1^2Q_1 \quad . \quad . \quad . \quad (13')$$

The values of S_3 and T_3 given in (12′), (13′) were verified by the use of the equations conjugate to (6).

8. We will now compute the correction to λ that is due to the terms of third order. From equation (18)

$$\left(\lambda^2 - \frac{n^2}{2}\right)^2 = \frac{n^4}{4}\left[1 - \frac{27m}{(1+m)^2} + \tfrac{1}{57}(P_1^2 + Q_1^2)\right].$$

If we extract the square root of both sides and expand the

right-hand member by the Binomial Theorem, we obtain, to the order of approximation used,

$$\lambda^2 - \frac{n^2}{2} = \frac{n^2}{2}\left[\left(1 - \frac{27m}{(1+m)^2}\right)^{\frac{1}{2}} + \tfrac{1}{2}\left(1 - \frac{27m}{(1+m)^2}\right)^{-\frac{1}{2}} \cdot \tfrac{1}{5^7} \cdot (P_1^2 + Q_1^2) + \cdots\right]$$

We take the positive sign of the radical because that sign corresponds to the value of λ that we are using. Hence

$$\lambda^2 - \frac{n^2}{2} = \frac{n^2}{2}(0{\cdot}987059 - 0{\cdot}029623 P_1 Q_1)$$

from which

$$\lambda = 0{\cdot}997236 - 0{\cdot}007443 P_1 Q_1 \quad . \quad . \quad (24b)$$

since $n = 1{\cdot}00048$.

9. To obtain the third approximation we equate to zero the coefficients of $e^{3\lambda ti}$ and $e^{-3\lambda ti}$ in the expression that we deduced by substituting (5) and (5*a*) in (3), retaining only the coefficients to the third order inclusive. This procedure gives us the equations

$$\left.\begin{aligned} N_3(9\lambda^2 + 6n\lambda + a_1) + b_1 M'_3 + X_3 &= 0 \\ M'_3(9\lambda^2 - 6n\lambda + a_1) + b_1 N_3 + Y_3 &= 0 \end{aligned}\right\} \quad . \quad . \quad (25)$$

where

$$X_3 = b_2 P_1 P_2 + b'_2(P_1 Q'_2 + Q_1 P_2) + c'_2 Q_1 Q'_2 + \tfrac{1}{3} a_3 P_1^3 + b_3 P_1^2 Q_1 + a_3 P_1 Q_1^2 + \tfrac{1}{3} c'_3 Q_1^3 \quad . \quad . \quad . \quad (26)$$

$$Y_3 = c_2 P_1 P_2 + b_2(P_1 Q'_2 + P_2 Q_1) + b'_2 Q_1 Q'_2 + \tfrac{1}{3} c_3 P_1^3 + a_3 P_1^2 Q_1 + b_3 P_1 Q_1^2 + \tfrac{1}{3} a_3 Q_1^3 \quad . \quad . \quad . \quad (27)$$

Equations (26), (27) enable us to compute X_3, Y_3. Then by means of (25) we can compute M'_3 and N_3. Four similar equations give us M_3 and N'_3. By assumption, M_3, M'_3, N_3, N'_3 are small quantities—on this assumption the whole method depends,—so that, in computing their values to the degree of approximation desired, we can take $m = 0$, $\lambda = n = 1$. Hence, making the substitutions of values already found for b_2, b'_2, c_2 . . . in terms of m and for P_2, Q_2, P'_2 . . . in terms of P_1 Q_1 in equations (26), (27), we have, after reducing,

$$X_3 = \tfrac{8}{6} P_1^2 Q_1$$
$$Y_3 = \tfrac{11}{6} P_1^2 Q_1 .$$

These values used in equations (25) give

$$M'_3 = -\tfrac{1}{9} P_1^2 Q_1 \quad . \quad . \quad . \quad . \quad (28)$$
$$N_3 = -\tfrac{8}{9} P_1^2 Q_1 \quad . \quad . \quad . \quad . \quad (29)$$

In a similar manner we easily obtain

$$X'_3 = \tfrac{11}{6} P_1^2 Q_1$$
$$Y'_3 = \tfrac{89}{6} P_1^2 Q_1$$

and

$$M_3 = -\tfrac{1}{9} P_1^2 Q_1 \quad . \quad . \quad . \quad . \quad (30)$$
$$N'_3 = -\tfrac{8}{9} P_1^2 Q_1 \quad . \quad . \quad . \quad . \quad (31)$$

10. We have next to compute P_3, P'_3, Q_3, Q'_3, the additions to the coefficients of $\sin \lambda t$ and $\cos \lambda t$ due to third order terms in the solution. Incidentally, we shall be able to test the value previously obtained for $\delta\lambda$.

In equations (6) we now replace Q'_1 by its equal Q_1. P_1 and Q_1, P'_1 and Q'_1 are real, but P_3, P'_3, Q_3, Q'_3, S_3, S'_3, T_3, T'_3 may be complex. In the first of equations (6) set $P_3 = p_3 + ip'_3$, $Q_3 = q_3 + iq'_3$, $S_3 = s_3 + is'_3$, and $\lambda = \lambda_0 + \delta\lambda$, where λ_0 is the original value of λ obtained in the first approximation and $\delta\lambda$ is the correction to be made to λ due to the third order terms. Here p_3, q_3, s_3, p'_3 . . . are real quantities and $i = \sqrt{-1}$. Obviously, the value of $\delta\lambda$ should agree with the value already found by another method in No. 9.

If we substitute the above values in the first equation of (6), we shall have

$$(P_1 + p_3 + ip'_3)[(\lambda_0 + \delta\lambda)^2 + 2n(\lambda_0 + \delta\lambda) + a_1] + b_1(Q_1 + q_3 + iq'_3) + s_3 + is'_3 = 0 \quad . \quad . \quad . \quad (a)$$

Three of the quantities p_3, p'_3, q_3 . . . are arbitrary, and to the required degree of accuracy we can, without loss of generality, set $p_3 = q'_3 = 0$. If we make this substitution in (a) and equate to zero separately the real and imaginary parts, and the first and second order terms, omitting terms of order higher than the third, we obtain

$$\left.\begin{aligned} \lambda_0^2 + 2n\lambda_0 + a_1 + \frac{Q_1}{P_1} b_1 &= 0 \\ 2\lambda_0 \delta\lambda + 2n\delta\lambda + \frac{q_3}{P_1} b_1 + \frac{s_3}{P_1} &= 0 \\ \frac{p'_3}{P_1}(\lambda_0^2 + 2n\lambda_0 + a_1) + \frac{s'_3}{P_1} &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad (32)$$

From (12′) we have

$$S_3 = (-19 + 29u_0 + \tfrac{3249}{64} m - \tfrac{1465}{32} m u_0) P_1^2 Q_1 .$$

But

$$S_3 = s_3 + is'_3$$

and

$$u_0 = e^{\frac{\pi i}{3}} = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} = \tfrac{1}{2} + \frac{i\sqrt{3}}{2}$$

Making these substitutions and equating the real and imaginary parts in (12′) give, if we make the necessary calculation,

$$s_3 = -4{\cdot}4734 P_1^2 Q_1$$
$$s'_3 = 25{\cdot}077 P_1^2 Q_1.$$

The use of this value of s'_3, with the value of n, λ_0, and a_1, already found, in the third equation of (32) will give

$$p'_3 = 16{\cdot}7264 P_1^3.$$

Similarly, if we make the substitutions $T_3 = t_3 + it'_3$

and $$s_0 = e^{-\frac{\pi i}{3}} = \cos\frac{\pi}{3} - i \sin\frac{\pi}{3} = \tfrac{1}{2} - \frac{i\sqrt{3}}{2}$$

in (13′) and as before equate the real and imaginary parts, we shall obtain

$$t_3 = -1{\cdot}5030 P_1^2 Q_1$$
$$t'_3 = 8{\cdot}3710 P_1^2 Q_1.$$

In a similar way we obtain from the second equation of (6)

$$\left(\frac{Q_1}{P_1} + \frac{q_3}{P_1}\right)(\lambda_0^2 + 2\lambda_0\delta\lambda - 2n\lambda_0 - 2n\delta\lambda + a_1)$$
$$+ \left(1 + \frac{ip'_3}{P_1}\right) b_1 + \frac{t_3 + it'_3}{P_1} = 0 \quad . \quad . \quad . \quad (a')$$

Equating to zero separately the real and imaginary parts of (a′) and also the first and second order terms, we have

$$\left.\begin{aligned} &\frac{Q_1}{P_1}(\lambda_0^2 - 2n\lambda_0 + a_1) + b_1 = 0 \\ &\frac{Q_1}{P_1}(2\lambda_0 - 2n)\delta\lambda + \frac{q_3}{P_1}(\lambda_0^2 - 2n\lambda_0 + a_1) + \frac{t_3}{P_1} = 0 \\ &\frac{p'_3}{P_1} b_1 + \frac{t'_3}{P_1} = 0 \end{aligned}\right\} \quad . \quad (32a)$$

If we use the values of t'_3 and b_1 already found in the third equation of (32a), we obtain

$$p'_3 = 16{\cdot}7259 P_1^3.$$

Now, by using the second equations of (32), (32a) and making the proper substitutions for known quantities and reducing, we have

$$\delta\lambda = -0{\cdot}00741 P_1 Q_1.$$

This value of $\delta\lambda$ agrees with the value found in No. 9 to the order of approximation used, it being remembered that we have omitted the second and higher powers of m.

From the second equation of (32) we easily deduce

$$q_3 = -8{\cdot}9973 P_1^3 .$$

If, now, we make use of the expressions just obtained for ρ'_3 and q_3, we have, since $P_3 = ip'_3$, $Q_3 = q_3$,

$$P_1 + P_3 = P_1 + i16{\cdot}726 P_1^3 \quad . \quad . \quad . \quad (33)$$

$$Q_1 + Q'_3 = -2{\cdot}9957 P_1 - 8{\cdot}997 P_1^3 \quad . \quad . \quad (34)$$

By means of the equations conjugate to (6) we deduce in precisely the same way as before

$$P_1 + P'_3 = P_1 - i16{\cdot}726 P_1^3 \quad . \quad . \quad . \quad (35)$$

$$Q_1 + Q_3 = -2{\cdot}9957 P_1 - 8{\cdot}997 P_1^3 \quad . \quad . \quad (36)$$

11. If, now, we substitute in (5), (5*a*) the values of the coefficients that we have computed and put $u_1 = x_1 + y_1 i$, $s_1 = x_1 - y_1 i$, $e^{n\lambda t i} = \cos n\lambda t + i \sin n\lambda t$ and equate real and imaginary parts, we obtain

$$\begin{aligned} x_1 = {} & (-1{\cdot}9957 \cos \lambda t) P_1 \\ & + (1{\cdot}9994 \cos 2\lambda t - 0{\cdot}0031 \sin 2\lambda t - 1{\cdot}9949) P_1^2 \\ & + (-16{\cdot}726 \sin \lambda t - 8{\cdot}997 \cos \lambda t + 2{\cdot}9957 \cos 3\lambda t) P_1^3 \\ & + \cdots \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37) \end{aligned}$$

$$\begin{aligned} y_1 = {} & (3{\cdot}9957 \sin \lambda t) P_1 \\ & + (0{\cdot}9964 \sin 2\lambda t - 0{\cdot}0097 \cos 2\lambda t + 8{\cdot}3554) P_1^2 \\ & + (16{\cdot}726 \cos \lambda t + 8{\cdot}997 \sin \lambda t + 2{\cdot}3299 \sin 3\lambda t) P_1^3 \\ & + \cdots \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38) \end{aligned}$$

12. The values of x_1 and y_1 were computed for twenty-three values of t and curves were drawn to scale for five values of the arbitrary parameter P_1, viz., $P_1 = \frac{1}{20}, \frac{1}{10}, \frac{1}{8}, \frac{1}{6}, \frac{1}{4}$.

These curves are shown on the accompanying plate (Plate 18). The axis of x_1 points nearly to the Sun, being directed away from it by an angle equal to about 1′ 25″. On the scale of the drawing it was not possible to represent this angle except by placing the centre of the Sun a trifle to the right of the x_1-axis. The scale to which the orbits were drawn is indicated on the drawing, as is also the direction of motion of the infinitesimal body, which is clockwise relative to the moving axes.

13. We will now determine the value of $\frac{1}{\lambda}$, which is the ratio of the period of the asteroid to that of Jupiter. We have found above that

$$\begin{aligned} \lambda &= 0{\cdot}9972 - 0{\cdot}00742 P_1 Q_1 \\ &= 0{\cdot}9972 + 0{\cdot}0222 P_1^2 \end{aligned}$$

since

$$Q_1 = -2{\cdot}9957 P_1 .$$

Making use of the above expression for λ, we obtain for the various orbits

P_1.	$\frac{1}{\lambda}$.
$\frac{1}{20}$	1·0027
$\frac{1}{10}$	1·0026
$\frac{1}{8}$	1·0025
$\frac{1}{6}$	1·0022
$\frac{1}{4}$	1·0014.

That is, the period decreases with the increase of P_1, and is a little longer than the period of Jupiter.

14. In order to determine the constant of relative energy, C, we make use of the Jacobian Integral

$$\dot{x}^2 + \dot{y}^2 = 2\Omega - C,$$

where

$$2\Omega = r^2 + \frac{2}{r} + m\left(\rho^2 + \frac{2}{\rho}\right) \quad \text{and} \quad m = \tfrac{1}{1048}.$$

We are going to compute the value of C for four points on each orbit. It is necessary first to obtain the value of $V^2 = \dot{x}_1^2 + \dot{y}_1^2$, and these values are given in a table below.

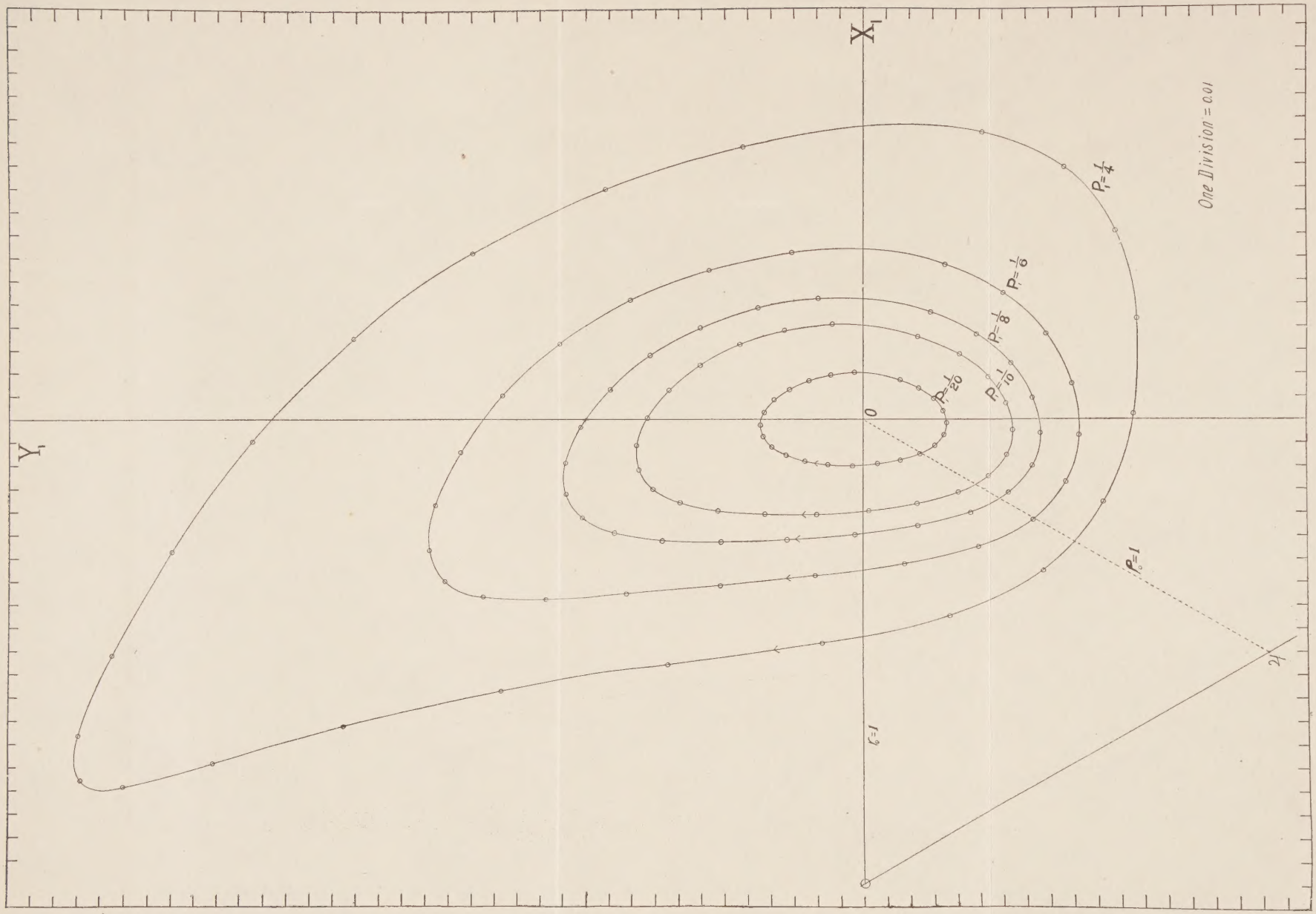
If we differentiate with respect to t the expressions for x_1 and y_1 and square and add, we obtain

$$\begin{aligned}\dot{x}_1^2 + \dot{y}_1^2 = \lambda^2[&(9{\cdot}9742 + 5{\cdot}9914\cos 2\lambda t)P_1^2 \\ &+ (0{\cdot}0899\sin\lambda t - 0{\cdot}0178\cos\lambda t \\ &+ 15{\cdot}943\cos 3\lambda t + 0{\cdot}0651\sin 3\lambda t)P_1^3 \\ &+ (63{\cdot}8856 + 39{\cdot}8546\cos 4\lambda t \\ &+ 0{\cdot}0635\sin 4\lambda t - 100{\cdot}2122\sin 2\lambda t \\ &+ 27{\cdot}9872\cos 2\lambda t)P_1^4 + \cdots].\end{aligned}$$

From this expression we obtain the following values of V^2, those of λ^2 being obtained from equation (24*b*):—

t.	V^2.	$P_1=\frac{1}{20}$.	$P_1=\frac{1}{10}$.	$P_1=\frac{1}{8}$.	$P_1=\frac{1}{6}$.	$P_1=\frac{1}{4}$.
$\frac{\pi}{3\lambda}$	$\lambda^2(6{\cdot}9785P_1^2 - 15{\cdot}874P_1^3 - 56{\cdot}8765P_1^4)$	0·0150	0·0480	0·0638	0·0761	−0·0339
$\frac{7\pi}{4\lambda}$	$\lambda^2(9{\cdot}9742P_1^2 - 11{\cdot}3956P_1^3 + 124{\cdot}2432P_1^4)$	0·0242	0·1003	0·1630	0·3188	0·9280
$\frac{5\pi}{4\lambda}$	$\lambda^2(9{\cdot}9742P_1^2 + 11{\cdot}1764P_1^3 - 76{\cdot}1812P_1^4)$	0·0258	0·1028	0·1582	0·2688	0·4990
$\frac{5\pi}{6\lambda}$	$\lambda^2(12{\cdot}9699P_1^2 + 0{\cdot}1255P_1^3 + 144{\cdot}6831P_1^4)$	0·0331	0·1436	0·2371	0·4705	1·3739
	$\lambda^2 = 0{\cdot}99447 + 0{\cdot}0445P_1^2 =$	0·9946	0·9949	0·9952	0·9957	0·9972

Since $r^2 = x_1^2 + y_1^2$, $\rho^2 = (1 - x_1)^2 + y_1^2$, the corresponding values of Ω can be obtained from the expression just given by the use of the values (37), (38). The equation $C = 2\Omega - V^2$ then gives C. Owing to the limited degree of approximation, these values, instead of being the same at all points of an orbit, will differ. The


X1
Y1
0
One Division = 0.01
P1=1/4
P1=1/6
P1=1/8
P1=1/10
P1=1/20
P0=1
1-ρ
2ρ

computed results for the different points on the five orbits are shown in the following table:—

Values of C.

t.	$P_1=\frac{1}{20}$.	$P_1=\frac{1}{10}$.	$P_1=\frac{1}{8}$.	$P_1=\frac{1}{6}$.	$P_1=\frac{1}{4}$.
$\frac{\pi}{3\lambda}$	2·9927	2·9596	2·9399	2·9658	4·1309
$\frac{7\pi}{4\lambda}$	2·9928	2·9617	2·9368	2·8814	2·7490
$\frac{5\pi}{4\lambda}$	2·9930	2·9698	2·9619	2·9761	3·3240
$\frac{5\pi}{6\lambda}$	2·9930	2·9588	2·9283	2·8469	2·4498

15. The curves shown on the plate indicate the nature of the changes with an increase of their linear dimensions. For $P_1=\frac{1}{20}$ it will be seen that the orbit is almost symmetrical, rather closely resembling an ellipse, which is the curve of motion when the first power only of P_1 is considered. For $P_1=\frac{1}{10}$ the orbit rises relatively higher along the positive part of the y_1-axis, and develops its vertex at the left of the y_1-axis. Here the shape of the curve differs more from the ellipse. The vertex is more to the left and that part of the orbit is more elongated for $P_1=\frac{1}{8}$ and $P_1=\frac{1}{6}$. For $P_1=\frac{1}{4}$ the curve becomes much more unsymmetrical, the vertex is more pointed, is much farther along the y_1-axis and also farther to the left of it. The curvature, too, on the left-hand side of the orbit changes sign, and becomes in one place slightly concave toward the Sun. A study of the curves shows that as the arbitrary parameter increases, which corresponds to a greater velocity in the orbit, the curves become more and more unsymmetrical. It would seem that with these larger velocities the motion must be rather unstable. The computed curves seem to indicate that with larger velocities the asteroid would go farther out into space, and, if the velocity were great enough, that the character of the orbit would change and lose its periodic form by tending to become infinitely great.

The particular points on the orbits for which the constant C was computed were chosen because they seemed to be representative points. For these points C varies but little for the curves for which $P_1=\frac{1}{20}$, and $P_1=\frac{1}{10}$, which indicates that the curves well represent the function. The variation of C is greater for $P_1=\frac{1}{8}$ and $P_1=\frac{1}{6}$, but here it seems probable that the curves represent fairly well the behaviour of the function under the chosen conditions. For $P_1=\frac{1}{4}$, however, the series probably either are not convergent or, at least, converge so slowly that to the degree of approximation used the curve does not give an accurate representation of the function. This is indicated by the wide variation of C for this value of P_1 as well as by the fact that for the point on the orbit corresponding to $t=\frac{\pi}{3\lambda}$ the computed value of V^2 is negative.

Since, in general, C tends to decrease as the arbitrary, P_1, increases, it would seem probable that the values of C in the orbit for $P_1 = \frac{1}{4}$ are more nearly correct for the points for which $t = \frac{7\pi}{4\lambda}$ and $t = \frac{5\pi}{6\lambda}$ than for the other two points considered.

To obtain by this method a more accurate representation of the function for larger values of the arbitrary, we should have to go to higher approximations. Since it would be very tedious to carry the approximation further, probably it would be better to use some other method for determining the orbits for values of P_1 greater than $\frac{1}{6}$.

The writer wishes to extend his thanks to Prof. Ernest W. Brown of Yale University for many suggestions offered during the carrying out of this work and the preparation of the manuscript for publication.

Orono, Maine, U.S.A.:
1913 *February* 1.